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Communication and Coordination in Social Networks

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I model people in a coordination game who use a communication network to tell each other their willingness to participate. The minimal sufficient networks for coordination can be interpreted as placing people into a hierarchy of social roles or "stages": "initial adopters", then "followers", and so on down to "late adopters". A communication network helps coordination in exactly two ways: by informing each stage about earlier stages, and by creating common knowledge within each stage. We then consider two examples: first we show that "low dimensional" networks can be better for coordination even though they have far fewer links than "high dimensional" networks; second we show that wide dispersion of "insurgents", people predisposed toward participation, can be good for coordination but too much dispersion can be bad.

1. INTRODUCTION

Although collective action depends on both social structure and individual incentives, these integral aspects have been formalized separately, in the fields of social network theory and game theory. By considering them together, this paper engages the classic question of which structures are conducive to coordination and shows that structure and strategy are related in a mutually interesting way.

Here we consider a coordination game in which each person wants to participate only if others participate. Social structure is thought of as a communication network by which people tell each other their willingness to participate. Each person knows whether her neighbours in the network are willing, but does not know about anyone else. Our model is thus a game of incomplete information in which each person, given his local knowledge, decides whether to participate.

The main result is a characterization of the minimal sufficient networks for coordination in terms of a hierarchy of social roles or "stages": "initial adopters", then "followers", and so on down to "late adopters", for example. A communication network helps coordination in exactly two ways: by informing each stage about earlier stages, and by creating common knowledge within each stage. We also consider two simple examples: first, we show that low dimension or "strong link" networks can be better for coordination even though they have far fewer links than high dimension or "weak link" networks; second, we show that wide dispersion of "insurgents", people predisposed toward participation, can be good for coordination but too much dispersion can be bad.

This paper is built on the assumption that the most basic and common mechanism for coordination is the "I'll go if you go" mechanism: individuals first communicate with each other about their preferences, and then each individual chooses whether to participate or not. Hence a person's participation depends on what he knows about his neighbours; it does not depend solely on their participation itself, either through learning, adaptation, or social influence. The physical analogies of contagion or diffusion are not really appropriate; the social network "carries" information, not influence or participation itself.

Although the model of course applies to coordination problems generally, its main motivation is collective political action, such as protest, revolt, or revolution. The importance of social networks for political participation has been explored in many contexts (for example Snow, Zurcher and Ekland-Olson (1980), and Opp and Gern (1993)) and is most clearly demonstrated by the efforts governments take to restrict them: for example, in the former East Germany "only about 13 percent of households had a telephone and there were few restaurants or pubs" (Opp and Gern (1993), p. 662); plantation owners in Hawaii around 1900 tried to discourage labour action by conscripting workers who spoke different native languages (Takaki (1983)). A combined structural and strategic analysis of collective action goes back at least to Marx, who saw the proletariat's emergence as a collective actor as resulting from machine production both concentrating workers in factories (structural) and reducing and levelling wages (strategic); interestingly, Marx emphasized improved communication networks resulting from technologies such as the railroad.

2. THE MODEL

Start with a finite set of people $N=\{1,2,\ldots,n\}$. Each person $i\in N$ chooses an action $a_i\in\{r,s\}$, where r is "revolt", the "risky" action, and s is "stay at home", the "safe" action. Each person i is either willing w or unwilling x. Person i has utility function $u_i\colon\{w,x\}\times A\to\Re$, where $A=\{r,s\}^n$. If person i is willing, then his willingness to revolt increases the more other people revolt: if $a,a'\in A$ and $a'_j=r\Rightarrow a_j=r$, then $u_i(w,r,a_{N\setminus\{i\}})-u_i(w,s,a_{N\setminus\{i\}})\geqq u_i(w,r,a'_{N\setminus\{i\}})-u_i(w,s,a'_{N\setminus\{i\}})$; in other words, his utility is "supermodular". We also assume that $u_i(w,r,r_{N\setminus\{i\}})\geqq u_i(w,s,r_{N\setminus\{i\}})$; in the best case in which everyone else revolts, a willing person wants to revolt. If person i is unwilling, then he always wants to stay at home: $u_i(x,a)=0$ if $a_i=r$ and $u_i(x,a)=1$ if $a_i=s$.

The communication network is a binary relation $\to$ on N, where $j\to i$ means that person j talks to person i. The network $\to$ itself is common knowledge, and we assume $i\to i$ throughout. Each person can either be willing or unwilling; we assume that person i knows about only his neighbours in the network, the people in his "ball" $B(i)=\{j\in N\colon j\to i\}$. The set of states of the world is $\Theta=\{w,x\}^n$. So when the actual state of the world is $\theta\in\Theta$, person i knows only that the state of the world is in the set $P_i(\theta)=\{(\theta_{B(i)},\phi_{N\setminus B(i)})\colon \phi_{N\setminus B(i)}\in\{w,x\}^{n-\#B(i)}\}$. Taken together the sets $\{P_i(\theta)\}_{\theta\in\Theta}$ form a partition of Θ, which we call $\mathscr{P}_i$. A strategy for person i is a function $f_i\colon\Theta\to\{r,s\}$ which is measurable with respect to $\mathscr{P}_i$, that is, for all $\theta,\theta'\in\Theta$, if $\theta,\theta'\in P$, where $P\in\mathscr{P}_i$, then $f_i(\theta)=f_i(\theta')$. The idea here is that if two states are in the same element of the partition $\mathscr{P}_i$, then person i cannot distinguish between them, and hence must take the same action in either state.

Say that F_i is the set of all strategies for person i, and let $F=\times_{i\in N}F_i$. Given $f\in F$, person i's *ex ante* expected utility is $EU_i(f)=\sum_{\theta\in\Theta}\pi(\theta)u_i(\theta_i,f(\theta))$, where prior beliefs are given by $\pi\in\Delta\Theta$. We say that f is an equilibrium if for all $i\in N$, and for all $g_i\in F_i$, we have $EU_i(f)\geqq EU_i(g_i,f_{N\setminus\{i\}})$; each person i is playing a best response given what everyone else is doing.

We call this game $\Gamma(\to,\pi)$. Because utilities are supermodular, it is possible to show that an equilibrium always exists.

Lemma 1. *An equilibrium of* $\Gamma(\to,\pi)$ *exists.*

3. MINIMAL SUFFICIENT NETWORKS

Which networks $\rightarrow$ enable the group to revolt? It depends on prior beliefs π. For example, if each person has a strong enough belief that everyone else is willing, then each person will revolt even if everyone is completely isolated and does not actually know that anyone else is willing. In other words, if everyone is already sufficiently optimistic, then revolt will take place regardless of the communication network. In this paper we take a more "pessimistic" or "conservative" view and say that a communication network is sufficient if it enables the group to revolt regardless of the prior π. For everyone to revolt, everyone must be willing, and hence we need to look only at that state of the world.

Definition. We say that $\rightarrow$ is a *sufficient network* if for all $\pi \in \Delta\Theta$, there exists an equilibrium f of $\Gamma(\rightarrow, \pi)$ such that $f_i(w, \ldots, w) = r$ for all $i \in N$.

For a network to enable revolt regardless of the prior, it must do so in the "worst case" in which any potential loss from revolting outweighs all possible gain: for example, if prior beliefs place almost all weight on people being unwilling (making losses extremely likely) or if a person receives an extremely large penalty if she revolts and not enough people join in (making losses extremely large). In the worst case, in equilibrium a person revolts only when there is absolutely no risk in doing so. In other words, $f_i(\theta) = r$ if and only if $\theta_i = w$ and $u_i(w, r, f_{N\setminus\{i\}}(\phi)) \geqq u_i(w, s, f_{N\setminus\{i\}}(\phi))$ for all $\phi \in P_i(\theta)$; a person revolts only if she is willing and gains in all states of the world which she knows might happen. The following lemma is helpful.

Lemma 2. *Let* $\underline{F} = \{f \in F: f_i(\theta) = r$ *if and only if* $\theta_i = w$ *and* $u_i(w, r, f_{N\setminus\{i\}}(\phi)) \geqq u_i(w, s, f_{N\setminus\{i\}}(\phi))$ *for all* $\phi \in P_i(\theta)\}$. *Then* $\rightarrow$ *is a sufficient network if and only if there exists* $f \in \underline{F}$ *such that* $f_i(w, \ldots, w) = r$ *for all* $i \in N$.

So one way of understanding our sufficient network definition is in terms of robustness: a sufficient network must enable revolt regardless of the prior. Another way is in terms of an implicit assumption that people revolt only when there is no risk in doing so. Anyhow, our definition allows us to focus attention on the communication network and leave the prior unspecified.

It is easy to show that more communication never hurts revolt. Say $\rightarrow \subset \rightarrow'$ if $i \rightarrow j \Rightarrow i \rightarrow' j$ for all $i, j \in N$. If $\rightarrow$ is sufficient and $\rightarrow \subset \rightarrow'$, then $\rightarrow'$ is also sufficient.

Lemma 3. *If* $\rightarrow \subset \rightarrow'$ *and* $\rightarrow$ *is a sufficient network, then* $\rightarrow'$ *is a sufficient network.*

Do sufficient networks exist? If $\rightarrow$ is the "complete network" ($i \rightarrow j$ for all $i, j \in N$), everyone talks to everyone else, and when everyone is willing, that fact is common knowledge; hence everyone revolting is an equilibrium regardless of the prior, and the complete network is sufficient. In general, however, the complete network is much more communication than is necessary. We say that $\rightarrow$ is a minimal sufficient network if it is sufficient and does not contain any smaller sufficient network.

Definition. We say that $\rightarrow$ is a *minimal sufficient network* if (1) $\rightarrow$ is a sufficient network and (2) if $\rightarrow'$ is a sufficient network and $\rightarrow' \subset \rightarrow$, then $\rightarrow' = \rightarrow$.

Minimal sufficient networks are exactly what is required for revolt; anything less is insufficient, and anything more is unnecessary. The main result of this paper is a characterization of minimal sufficient networks. All minimal sufficient networks can be thought of as a hierarchy of "cliques". A clique of $\rightarrow$ is a set $M_k \subset N$ such that $i \rightarrow j$ for all $i, j \in M_k$, that is, a subgroup in which each person talks to everyone else.

Proposition. *Say $\rightarrow$ is a minimal sufficient network. Then there exist cliques $M_1, M_2, \ldots, M_z$ which cover N and a binary relation $\rightarrow$ defined over $M_1, M_2, \ldots, M_z$ such that* (1) *$i \rightarrow j$ if and only if there exists some M_k which contains i and M_l which contains j such that $M_k \rightarrow M_l$ and* (2) *if $M_{i_{y-1}} \rightarrow M_{i_y}$, then there exists a totally ordered set $M_{i_1}, M_{i_2}, \ldots, M_{i_{y-1}}, M_{i_y}$, where M_{i_1} is maximal.*

The cliques $M_1, M_2, \ldots, M_z$ cover N (that is, $N = M_1 \cup \cdots \cup M_z$): in general they do not partition N, and so a person can belong to more than one clique. Fact (1) says there exists a relation between these cliques, which we also designate by $\rightarrow$, which completely determines relations between individuals: whether I talk to you is determined completely by whether I belong to a clique which "collectively" talks to a clique you belong to. Fact (2) says that these cliques are arranged hierarchically in the sense that a relation between any two cliques is part of a totally ordered set of cliques ($M_{i_1}, M_{i_2}, \ldots, M_{i_y}$, where $k < l \Rightarrow M_{i_k} \rightarrow M_{i_l}$) in other words, each relation among cliques is part of a "chain" which starts with a "leading clique", then has cliques following in sequence, in which each clique knows about earlier cliques.

Coincidentally, sociology's "social network theory" has a standard interpretation of this result (for a survey, see Wasserman and Faust (1994), chapter 10). Each clique is a social role: relations between individuals can be described completely by relations between social roles. Relations among roles are interpreted as a hierarchy among social roles. Thus we call the cliques $M_1, M_2, \ldots, M_z$ and $\rightarrow$ defined on these cliques a *social role hierarchy*.

For example, consider the "threshold game" $\Gamma_{e_1,\ldots,e_n}$, where person i wants to revolt if at least e_i people revolt (in other words, $u_i(w, a) = 1$ if $a_i = r$ and $\#\{j \in N: a_j = r\} \geq e_i$, $u_i(w, a) = -1$ if $a_i = r$ and $\#\{j \in N: a_j = r\} < e_i$, and $u_i(w, a) = 0$ if $a_i = s$). In the threshold game $\Gamma_{2,2,4,4}$, there is a unique minimal sufficient network, shown in Figure 1; the threshold 2 people are the "leading adopters" and the threshold 4 people are the "followers" (for clarity, arrowheads are not drawn when a link is bidirectional).

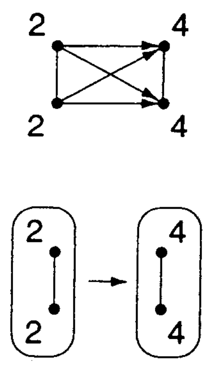


FIGURE 1

Minimal sufficient network of $\Gamma_{2,2,4,4}$ and its corresponding social role hierarchy

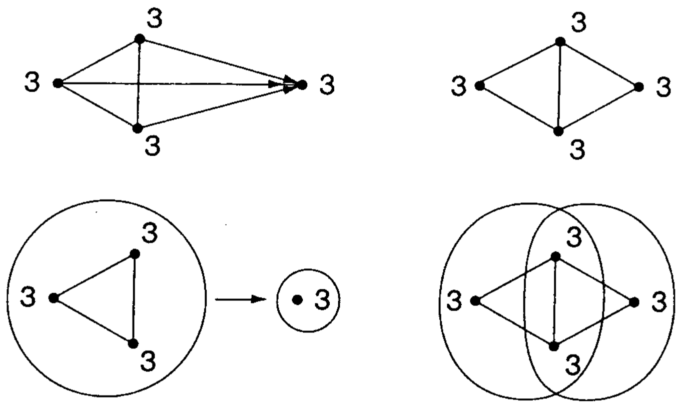


FIGURE 2

Two minimal sufficient networks of $\Gamma_{3,3,3,3}$ and corresponding social role hierarchies

In the threshold game $\Gamma_{3,3,3,3}$, there are two minimal sufficient networks, shown in Figure 2; in the first, there is a leading clique of three people and a single follower, and in the second there are two overlapping leading cliques.

Figure 3 shows a social role hierarchy for the game $\Gamma_{1,3,3,4,4,4,4,6,6,9,9,9}$. Here there are two leading cliques: the threshold 1 person and the four threshold 4 people. Note that the threshold 9 people need to know about the threshold 1 person not because they rely on his participation directly but because the threshold 3 people rely on him.

To see why cliques are important, consider again the game $\Gamma_{3,3,3,3}$ and the "square" network in Figure 4. Intuitively, say I am in this network and considering whether to revolt; since I know that there are three people with threshold 3 who are willing (my neighbours and myself), I know that there is sufficient discontent to make revolt possible.

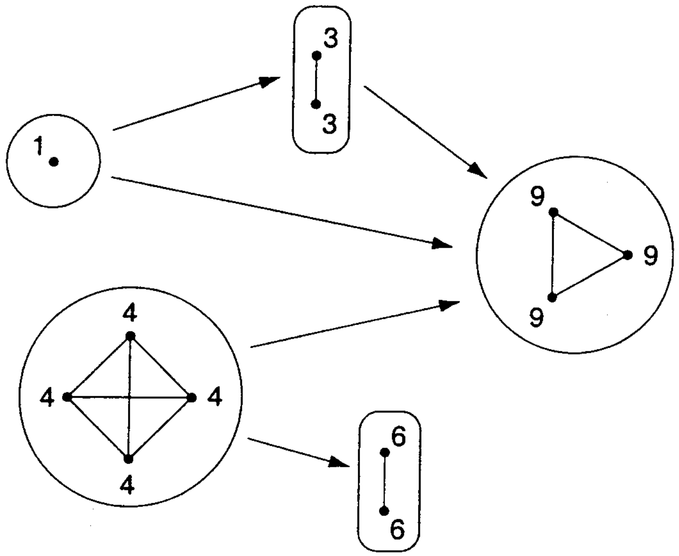


FIGURE 3

A social role hierarchy of $\Gamma_{1,3,3,4,4,4,4,6,6,9,9,9}$

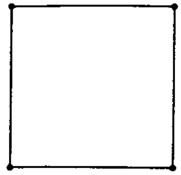


FIGURE 4
The square

But do I actually revolt? I know that my neighbour on the right, for example, is willing and I know that he knows I am willing. But I do not know whether his other neighbour ("across" from me) is willing, and hence I cannot count on him revolting. Since I cannot count on him (or anyone else), I do not revolt. So even though everyone in the square knows that revolt is possible, no one knows that anyone also knows, and hence no one in fact revolts. In a three person clique, as shown in Figure 2, however, each individual not only knows that his two neighbours are willing, but also that they know that each other are willing.

The model shows how common knowledge comes up naturally in a network context (see also Morris (1997)); a clique is the graphical representation of "local" common knowledge. The importance of common knowledge for coordination has been demonstrated in a variety of strategic contexts (for example Chwe (1999*a*), Morris, Rob and Shin (1995), Rubinstein (1989), Shin (1996)) and was first discussed by Lewis (1969).

Cliques must not only form; information must also "flow" starting from "leading cliques" through chains of cliques. This is reminiscent of Schelling's (1978) "chain reaction" model, in which each person decides to participate after observing people already participating. In our model, people know each others' willingness, and do not observe actions. Still, the models are similar in illustrating how coordination can occur through unidirectional communication.

Minimal sufficient networks are a game's "inherent structures", interpretable as hierarchical social roles. Communication helps in coordination in exactly two ways: by creating common knowledge inside each social role and by informing each social role about its predecessors. Lewis's common knowledge formation and Schelling's unidirectional communication were among the first coordination mechanisms to be explicitly described. The proposition suggests that these two communication mechanisms are a "basis" for all communication networks: a communication network is successful exactly to the extent that it combines these two mechanisms in an appropriate way.

4. STRONG LINKS, WEAK LINKS, AND DIMENSION

In this example we consider the "dimension" of a large population playing a threshold game. We show that low dimensions can be better for revolt than high dimensions, even though in low dimensions each person knows far fewer people. This distinction between low and high dimension is exactly sociology's distinction between strong and weak links (Granovetter (1973); for more examples see Chwe (1999*b*)).

Say we have the threshold game $\Gamma_{e,\dots,e}$, in which each person wants to revolt as long as e people in total revolt, and say the network $\rightarrow$ is symmetric. For person i to revolt, it must be that he belongs to a minimal sufficient network. By the proposition, this minimal sufficient network has a "leading clique" M_k such that $j \rightarrow i$ for all $j \in M_k$. Since $\rightarrow$ is

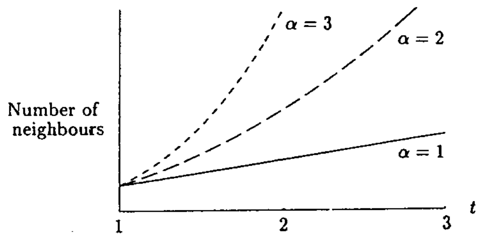


FIGURE 5
Number of neighbours each person has in $\rightarrow^t$ for dimensions $\alpha = 1, 2, 3$

symmetric, person i belongs to the leading clique M_k. Since each person has a threshold of e, it is easy to see that the leading clique must have at least e members. So our game-theoretic model yields a simple graph-theoretic result: when everyone has threshold e and the network is symmetric, revolt is possible if and only if one is in a clique of willing people of size e.

Say that a large population of willing people live on a set H contained in $\Re^m$. The distance between two points $x, y \in \Re^m$ is defined to be $d(x, y) = \max_{i=1}^m |x_i - y_i|$; the set of points within distance q of x is the "closed ball" $b_q(x) = \{y \in \Re^m : d(x, y) \leq q\}$. People are distributed randomly and evenly on the set H: we assume that for all $x \in H$, the expected number of people who live in $b_q(x)$ is given by βq^α, where $\beta > 0$ is a constant and α is the dimension of H. For example, if H is a line, it has dimension $\alpha = 1$; if H is a plane, it has dimension $\alpha = 2$.

Say that the communication network grows as time progresses. Say two people j and k are connected at time t if they are within distance t of each other; that is, if person j lives at x, person k lives at y, and $d(x, y) \leqq t$, then we write $j \rightarrow^t k$. We start at time $t = 1$. Figure 5 plots βt^α, the expected number of neighbours a person has, for dimensions $\alpha = 1, 2, 3$. High dimension networks are denser in that each person has more neighbours.

We know that a person can revolt if and only if he is in a $\rightarrow^t$-clique of size e. But given our distance function d, it is not hard to show that the cliques of $\rightarrow^t$ are simply those people who live in some $b_{t/2}(x)$, a closed ball of radius $t/2$. Each clique contains on average $\beta(t/2)^\alpha$ people; hence revolt is possible once $\beta(t/2)^\alpha$ is greater than e. Figure 6 plots $\beta(t/2)^\alpha$, the size of a clique at time t, for dimensions $\alpha = 1, 2, 3$.

For low thresholds e, the low dimension networks form cliques of size e faster and hence are better for revolt; for high thresholds, higher dimensions are better. But notice

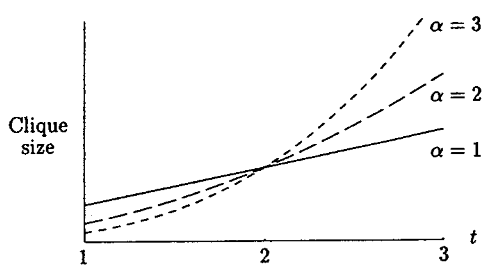
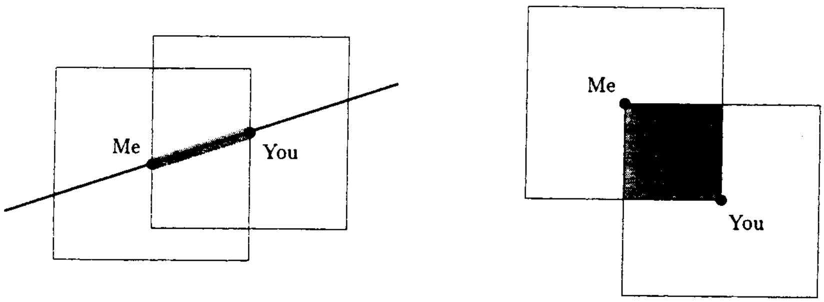


FIGURE 6
Size of cliques of $\rightarrow^t$ for dimensions $\alpha = 1, 2, 3$



$\alpha = 1$ $\alpha = 2$

FIGURE 7

When $\alpha = 1$, at least half of your friends are friends of mine; when $\alpha = 2$, at least one fourth of them are

that when $t < 2$, low dimension networks have larger cliques even though each person has fewer neighbours. How is this possible? Figure 7 shows how: say that you are my neighbour. In a line, at least half of your neighbours are also neighbours of mine; in a plane, possibly only one fourth of your neighbours are my neighbours; in dimension 3, this fraction goes down to 1/8, and so on. The more often that neighbours of my neighbours are also my neighbours, the "greater the transitivity" of the network, the easier it is for cliques to form. Low dimension networks have greater transitivity, but high dimension networks have many more links. To form small cliques, transitivity, "involutedness", dominates and so lower dimensions are better. To form large cliques, the greater "connectivity" of higher dimensions is the overwhelming factor.

From actual network data, Rapoport and Horvath (1961) plot a graph like Figure 5, called a "tracing" in social network theory. They start with an arbitrary person, find two of her close friends, then find two close friends of each of these two people, and so forth. With each iteration, the group increases slowly because often no one new is added: the close friends of my close friends tend to be my close friends also. When instead they successively add two acquaintances, the group grows quickly: acquaintances of my acquaintances tend not to be my acquaintances. Thus Granovetter (1973) called links which tend to be transitive "strong" links and links which tend to scatter widely "weak" links. In our terminology, strong links have low dimension and weak links have high dimension. Strong links traverse a society slowly, while weak links are "fast": a demonstration suggests that any two people in the United States can be connected by as few as six weak links (Milgram (1992)). To connect a large society, then, weak links are more important than strong links; weak links are also better for the diffusion of information (Granovetter (1995); see also Montgomery (1991) and Finnernan and Kelly (1997)).

For collective political action, however, the importance of strong versus weak links is unclear. For example, data from volunteers in the 1964 Mississippi Freedom Summer, in which college students went into the southern states of the U.S. to protest against racial segregation, show that the presence of a strong link to another potential participant correlates strongly and positively with participation while the presence of a weak link has no correlation (McAdam (1986), McAdam and Paulsen (1993)). McAdam (1986, p. 80) interprets this as the links having different functions: "although weak links may be more effective as diffusion channels, strong ties embody greater potential for influencing behavior". This is of course reasonable, but our model suggests it is not necessary. If you and I are potential participants connected by a strong link, your friends are likely to be my friends, and participation among our group of friends would be common knowledge among us. If you and I are connected by a weak link, your friends and my friends do not know each other. Strong links are better for forming common knowledge at a local level, and when thresholds are low, local mobilization is sufficient (see also Marwell and Oliver (1993)). In other words, the argument that weak links, in our example high dimensions, are always better for communication relies on the implicit assumption that a person does not care about what her friends know about each other.

5. EFFICIENT SEEDING AND OPTIMAL DISPERSION

Say we have a group of "conservatives" with threshold e who do not revolt. However, we can "seed" revolt by adding a small number of "insurgents", people with threshold 1. How can we trigger revolt among the conservatives with the fewest number of insurgents?

For simplicity, say that our set of people N is the integers and two people are connected if they are within distance δ of each other ($i \rightarrow j$ if $|i-j| \leqq \delta$). Note that everyone is in a clique of size $\delta+1$. Since we assume that the conservatives do not revolt unassisted, we assume that $e > \delta+1$. Note also that each person is connected to $2\delta+1$ people in total; hence if $e > 2\delta+1$, a conservative would not revolt even if everyone else was an insurgent. Hence we assume $e \leqq 2\delta+1$.

One minimal sufficient network is a clique of conservatives of size e, which does not form by assumption. All other minimal sufficient networks involve a clique of conservatives which is led by some insurgents. For example, one minimal sufficient network has a clique of $e-1$ conservatives which is led by one insurgent, another has a clique of $e-2$ conservatives led by two insurgents, and so on.

Say $\{i, i+1, \ldots, i+l\}$ is a clique of $l+1$ conservatives; hence $l \leqq \delta$. Say that we try to get these conservatives to revolt by giving them some insurgents to depend on. The set of people who are connected to every member of the clique, but are not actually in the clique, is the "fringe" $\{i+l-\delta, \ldots, i-1\} \cup \{i+l+1, \ldots, i+\delta\}$. Since the clique is of size $l+1$, we need to add $e-l-1$ insurgents to the fringe. The fringe itself contains $2(\delta-l)$ people. So as long as $e-l-1 \leqq 2(\delta-l)$, we can make the clique revolt by adding insurgents to the fringe. But notice that the larger l is, the fewer insurgents are necessary; to minimize the number of insurgents we need to add, we maximize l such that $e-l-1 \leqq 2(\delta-l)$ and get $l=2\delta+1-e$. Hence the way to mobilize conservatives with the fewest number of insurgents is to take a clique of $2\delta+2-e$ conservatives and surround them with $e-\delta-1$ insurgents on each side (by our assumptions, both of these numbers are at least 1). To mobilize more cliques of conservatives, one can simply repeat this pattern. Figure 8 shows some examples for $\delta=3$ and $e=5, 6, 7$. When $e=5$, a clique of three conservatives depends on the two adjacent insurgents; when $e=6$, a clique of two conservatives depends on the surrounding four insurgents; when $e=7$, a single conservative depends on six surrounding insurgents.

Note that when $e=6$ we have one insurgent for each conservative. If we keep this ratio but simply alter the spacing, what happens? As shown in Figure 9, if we alternate conservatives and insurgents, no conservatives revolt: the largest clique of conservatives has two people, and only three insurgents are connected to both of them. If we place insurgents and conservatives in groups of three, then again no conservatives revolt: the largest clique of conservatives has three people, and only two insurgents are connected to

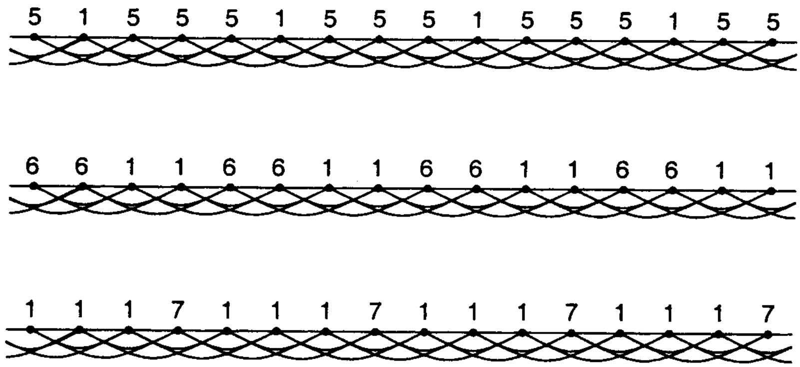


FIGURE 8
Efficient seeding

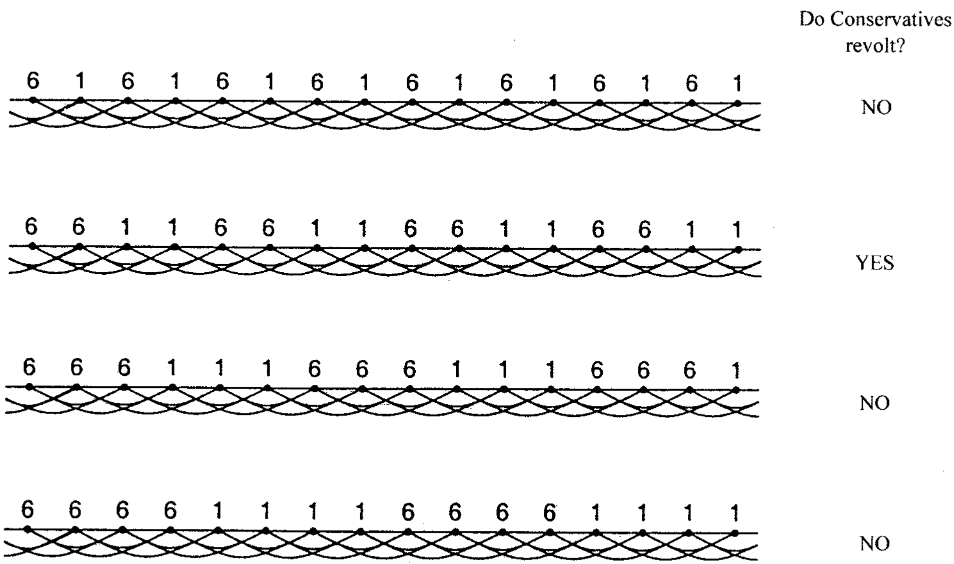


FIGURE 9
Optimal dispersion of insurgents

all of them. If we place insurgents and conservatives in groups of four, then no conservatives revolt because the largest clique of conservatives has four people, and no insurgents are connected to all of them.

Insurgents should be optimally dispersed: if they are dispersed too finely, they are "atomized"; if they are not dispersed enough, they are "ghettoized". This example naturally applies to "vanguard" party organization (Lenin (1969)) but also to marketing (Iacobucci and Hopkins (1992)): if people are more likely to use a new software programme if their friends use it, discount coupons, which lower thresholds, should neither be widely individually scattered nor concentrated all in one region, but should have many local concentrations.

6. DISCUSSION

Two of the model's assumptions are unimportant in the sense that relaxing them would not change much (of course, they might be crucial in a more general model). First, our assumption that communication is not strategic is not an issue because an unwilling person has no reason to say that he is willing, and a willing person would never say that he is unwilling. Second, it would be more reasonable to assume that each person only knows the network among his neighbours, as opposed to knowing the entire network. But since in effect you believe that people who are not your neighbours are almost certainly unwilling, it does not matter whether you know how they are connected.

The assumption that the network must enable coordination regardless of the prior is a strong one. If the prior were specified or bounded, then for example instead of requiring cliques, in which each person is connected to every other, one would require p-cliques, in which each person is connected to at least a proportion p of the others. The question of how much information is necessary for coordination has been pursued in more general contexts (for example Morris, Rob and Shin (1995), Kajii and Morris (1997)), with results in which conditions on prior beliefs play a crucial role. At least in the more limited context pursued here, with priors left unspecified and with information having the specific structure generated by a network, exact characterizations are obtainable.

Relaxing other assumptions might lead to some interesting new questions. First of all, in our model there is no disagreement over what to coordinate on: socialism versus social democracy, or VHS versus Beta, for example. If there were competing coordinations, the social network would influence what eventually gets chosen. Secondly, here either two people are linked or they are not. More generally, I might randomly talk with some people more often than others in a stochastic "contact process". Communication links might also be "noisy", from misunderstandings or imperfections in technology (Chwe (1995)). Our model also does not allow aggregate information: there is no way for someone to know that three others are willing without also knowing who these people are. Thirdly, we assume that the network is unchanging, but of course people intentionally make new acquaintances and even maintaining the existing network can be costly (Boorman (1975), Hendricks, Piccione and Tan (1995), Kelly (1997)). To say what a person gains by changing the network, however, one should be able to say what happens in the static cases of before and after (for example Aumann and Myerson (1985), Jackson and Wolinsky (1995), Watts (1997)). In the political context, a strategic model of network formation would include activists building organizations and creating links and governments repressing communication, destroying and restricting links. Finally, people perhaps should be able to communicate not only their willingness, but also the very fact of participation; if I see a very conservative person, or two socially distant people revolt, that would be strong evidence that many people are revolting.

Recently, social structure has come up in the context of "local interaction games", in which each person's payoff depends on the actions of his neighbours (for example Akerlof (1997), Anderlini and Ianni (1996), Blume (1995), Ellison (1993), Goyal (1996), Mailath, Samuelson and Shaked (1997), Morris (1999), Temzelides (1997), Young (1998)). Our model is a "local information game": locality is represented by information and not necessarily by payoffs (see also Bala and Goyal (1998)); of course since our coordination game is general, payoffs can be local also. Local interaction games make sense for local coordinations, such as keeping our street clean; for "big" coordinations such as political change, informational locality is more appropriate. Also, in a local interaction game, a person's actions depend only on the actions of his neighbours and not how those neighbours are connected. Under local interaction, for example, cliques do not have the same crucial role, optimal seeding would be as dispersed as possible, and the "slow neighbour growth" of low dimensions is in general better (Morris (1999)). The extent to which informational locality or payoff locality (or both) are important in a given situation is an empirical issue; a crude screening test for informational locality would be to see if participation is correlated not just with neighbour participation, but also with the presence of links between neighbours. For example, when Opp and Gern (1993) surveyed participants in the demonstrations which led to the collapse of East Germany, they simply asked each person whether he had friends who participated, and found that this was a significant variable in predicting his participation. Our model suggests that each person should also be asked if his friends who participated knew each other.

Theories of social structure, in sociology (for example Gould (1993), Macy (1991)) as well as economics, almost always make adaptive, bounded rationality, or behavioural assumptions. There seems little reason for this in principle; the technical complexities, however, of modelling complete rationality in a structural context might be a reason in practice. This paper shows that these complexities can be made manageable, resulting in some structural observations not otherwise discernible.

In game theory generally, social structure has come up in several contexts; for example, some models of bargaining (Myerson 1977) and coalition formation (Aumann and Myerson (1985), Kirman, Oddou and Weber (1986)) model social structure with a network or graph. In models of trade, local interactions were considered early on (for example Föllmer (1974), Kalai, Postlewaite and Roberts (1978), Allen (1982); more recently, Bell (1997), Kranton and Minehart (1997)). Networks also naturally come up in models of organizational hierarchies (for example Radner (1992) and Van Zandt (1997)). But social structure still is not a prominent concern of game theory or economic theory generally. Supported by entire scholarly traditions in sociology, the consideration of social structure offers greater contextualization and richness in understanding communication and coordination specifically and social phenomena more generally.

APPENDIX

Lemma 1. *An equilibrium of* $\Gamma(\rightarrow, \pi)$ *exists.*

Proof. This follows directly from the Knaster–Tarski fixed point theorem (Davey and Priestley (1990)): an order-preserving function on a lattice has a fixed point. We set up the lattice structure and show that the best response function is order-preserving; a fixed point of the best response function is an equilibrium.

Define the binary relation $\leqq$ on the set of strategy profiles F: we say $f\leqq g$ if $f_i(\theta)=r \Rightarrow g_i(\theta)=r$ for all $i\in N$ and all $\theta\in\Theta$. It is easy to see that $\leqq$ is transitive ($e\leqq f, f\leqq g \Rightarrow e\leqq g$) and antireflexive ($f\leqq g, g\leqq f \Rightarrow f=g$).

Given $E\subset F$, we define $(\bigvee_{f\in E} f)_i: \Theta\rightarrow\{r, s\}$ as $(\bigvee_{f\in E} f)_i(\theta)=r$ if $\exists f\in E$ such that $f_i(\theta)=r$ and $(\bigvee_{f\in E} f)_i(\theta)=s$ otherwise. It is easy to show that $(\bigvee_{f\in E} f)_i$ is measurable with respect to $\mathscr{P}_i$ and therefore $\bigvee_{f\in E} f\in F$.

Given $f\in F$, define $BR_i(f)(\theta)$ as person i's best response in state θ: we say $BR_i(f)(\theta)=r$ if $\theta_i=w$ and $\Sigma_{\phi\in P_i(\theta)}\pi(\phi)(u_i(w, r, f_{N\setminus\{i\}}(\phi))-u_i(w, s, f_{N\setminus\{i\}}(\phi)))\geqq 0$ and $BR_i(f)(\theta)=s$ otherwise. It is easy to show that $BR_i(f)$ is measurable with respect to $\mathscr{P}_i$ and hence $BR(f)\in F$.

Show that $BR: F\rightarrow F$ is order-preserving, that is, $f\leqq g \Rightarrow BR(f)\leqq BR(g)$. Say $f\leqq g$. Say $BR_i(f)(\theta)=r$ and $BR_i(g)(\theta)=r$. Since $BR_i(f)(\theta)=r$, we know that $\theta_i=w$ and $\Sigma_{\phi\in P_i(\theta)}\pi(\phi)(u_i(w, r, f_{N\setminus\{i\}}(\phi))-u_i(w, s, f_{N\setminus\{i\}}(\phi)))\geqq 0$. Since $f\leqq g$ and utilities are supermodular, we know that $\Sigma_{\phi\in P_i(\theta)}\pi(\phi)(u_i(w, r, g_{N\setminus\{i\}}(\phi))-u_i(w, s, g_{N\setminus\{i\}}(\phi)))\geqq 0$ and hence $BR_i(g)(\theta)=r$.

Let $E=\{f\in F: f\leqq BR(f)\}$. Let $b=\bigvee_{f\in E} f\in F$. It is easy to see that $f\leqq b$ for all $f\in E$. Since BR is order-preserving, we know $BR(f)\leqq BR(b)$ for all $f\in E$. It is easy to see that therefore $\bigvee_{f\in E} BR(f)\leqq BR(b)$. Since $f\leqq BR(f)$ for all $f\in E$, it is also easy to see that $\bigvee_{f\in E} f\leqq \bigvee_{f\in E} BR(f)$. Since $\leqq$ is transitive, we have $\bigvee_{f\in E} f\leqq$

$BR(b)$. But $\bigvee_{f\in E} f = b$. and thus $b \leqq BR(b)$. Since $b \leqq BR(b)$ and BR is order-preserving, we have $BR(b) \leqq BR(BR(b))$. Hence $BR(b) \in E$ and thus $BR(b) \leqq b$. Since $BR(b) \leqq b$ and $b \leqq BR(b)$, and $\leqq$ is antireflexive, we have $b = BR(b)$. Hence b is an equilibrium.

For later reference, we show that b is "increasing", that is, $\theta' \lesssim \theta \Rightarrow \{i \in N: b_i(\theta') = r\} \subset \{i \in N: b_i(\theta) = r\}$, where $\theta' \lesssim \theta$ means that $\theta'_i = w \Rightarrow \theta_i = w$. In other words, the more willing people, the more revolt. Define $c_i: \Theta \to \{r, s\}$ as $c_i(\theta) = r$ if there exists $\theta' \lesssim \theta$ such that $b_i(\theta') = r$ and $c_i(\theta) = s$ otherwise. It is also easy to show that c_i is measurable with respect to $\mathscr{P}_i$ and hence $c \in F$. It is easy to see that c is increasing, $b \leqq c$, and $c_i(\theta) = r \Rightarrow \theta_i = w$.

Show $c \leqq BR(c)$. Let $c_i(\theta) = r$. Hence $\theta_i = w$ and there exists $\theta' \lesssim \theta$ such that $b_i(\theta') = r$. It suffices to show that $\Sigma_{\phi\in P_i(\theta)} \pi(\phi)(u_i(w, r, c_{N\setminus\{i\}}(\phi)) - u_i(w, s, c_{N\setminus\{i\}}(\phi))) \geqq 0$ for all $\phi \in P_i(\theta)$. Let $\phi \in P_i(\theta)$. Hence $\phi = (\theta_{B(i)}, \phi_{N\setminus B(i)}) \lesssim (\theta'_{B(i)}, \phi_{N\setminus B(i)}) \in P_i(\theta')$. Since $b_i(\theta') = r$ and $b = BR(b)$, we know $\Sigma_{\phi\in P_i(\theta)} \pi(\phi)(u_i(w, r, b_{N\setminus\{i\}}(\theta'_{B(i)}, \phi_{N\setminus B(i)})) - u_i(w, s, b_{N\setminus\{i\}}(\theta'_{B(i)}, \phi_{N\setminus B(i)}))) \geqq 0$. Since $b \leqq c$ and utilities are supermodular, we know $\Sigma_{\phi\in P_i(\theta)} \pi(\phi)(u_i(w, r, c_{N\setminus\{i\}}(\theta'_{B(i)}, \phi_{N\setminus B(i)})) - u_i(w, s, c_{N\setminus\{i\}}(\theta'_{B(i)}, \phi_{N\setminus B(i)}))) \geqq 0$. Since c is increasing and utilities are supermodular, we know $\Sigma_{\phi\in P_i(\theta)} \pi(\phi)(u_i(w, r, c_{N\setminus\{i\}}(\phi)) - u_i(w, s, c_{N\setminus\{i\}}(\phi))) \geqq 0$.

Since $c \leqq BR(c)$, we have $c \in E$ and hence $c \leqq b$. But $b \leqq c$ and therefore $b = c$. Thus b is increasing. ||

Lemma 2. *Let* $\underline{F} = \{f \in F: f_i(\theta) = r$ *if and only if* $\theta_i = w$ *and* $u_i(w, r, f_{N\setminus\{i\}}(\phi)) \geqq u_i(w, s, f_{N\setminus\{i\}}(\phi))$ *for all* $\phi \in P_i(\theta)\}$. *Then* $\to$ *is a sufficient network if and only if there exists* $f \in \underline{F}$ *such that* $f_i(w, \ldots, w) = r$ *for all* $i \in N$.

Proof. ($\Rightarrow$) Let $\pi(\theta) = p(\theta_1) \cdots p(\theta_n)$, where $p(w) = \varepsilon$ and $p(x) = 1 - \varepsilon$. Let $z = \max\{u_i(w, r, a_{N\setminus\{i\}}) - u_i(w, s, a_{N\setminus\{i\}}): a \in A, i \in N\}$ and let $y = \max\{u_i(w, r, a_{N\setminus\{i\}}) - u_i(w, s, a_{N\setminus\{i\}}): u_i(w, r, a_{N\setminus\{i\}}) - u_i(w, s, a_{N\setminus\{i\}}) < 0, a \in A, i \in N\}$ (if $u_i(w, r, a_{N\setminus\{i\}}) - u_i(w, s, a_{N\setminus\{i\}})$ is never negative, then everyone has a weakly dominant strategy of revolting when willing, and the lemma holds trivially). Since $y < 0$, we can pick $\varepsilon > 0$ such that $(1-\varepsilon)^n y + (1 - (1-\varepsilon)^n) z < 0$.

Since $\to$ is sufficient, we know that there exists an equilibrium f of $\Gamma(\to, \pi)$ such that $f_i(w, \ldots, w) = r$ for all $i \in N$. By the proof of Lemma 1, there exists an increasing equilibrium b such that $f \leqq b$ and hence $b_i(w, \ldots, w) = r$ for all $i \in N$. It suffices to show that $b \in \underline{F}$. We need to show that if $b_i(\theta) = r$, then $\theta_i = w$ and $u_i(w, r, b_{N\setminus\{i\}}(\phi)) \geqq u_i(w, s, b_{N\setminus\{i\}}(\phi))$ for all $\phi \in P_i(\theta)$ (the converse is obvious). Say that $b_i(\theta) = r$. Obviously $\theta_i = w$. Say that there exists some $\phi = (\theta_{B(i)}, \phi_{N\setminus B(i)}) \in P_i(\theta)$ such that $u_i(w, r, b_{N\setminus\{i\}}(\phi)) < u_i(w, s, b_{N\setminus\{i\}}(\phi))$. Then if we let $\phi^* = (\theta_{B(i)}, x_{N\setminus B(i)}) \in P_i(\theta)$, since $\phi^* \lesssim \phi$ (fewer people are willing in ϕ^* than in ϕ), b is increasing, and utilities are supermodular, we have $u_i(w, r, b_{N\setminus\{i\}}(\phi^*)) - u_i(w, s, b_{N\setminus\{i\}}(\phi^*)) < 0$. We know $\Sigma_{\phi\in P_i(\theta)} \pi(\phi) = \varepsilon^{k_w}(1-\varepsilon)^{k_x}$, where $k_w = \#\{j \in B(i): \theta_j = w\}$ and $k_x = \#\{j \in B(i): \theta_j = x\}$. We also know that $\pi(\phi^*) = \varepsilon^{k_w}(1-\varepsilon)^{n-k_w}$. Hence

$$\Sigma_{\phi\in P_i(\theta)} \pi(\phi)(u_i(r, b_{N\setminus\{i\}}(\phi)) - u_i(s, b_{N\setminus\{i\}}(\phi))) \leqq \pi(\phi^*)y + (\varepsilon^{k_w}(1-\varepsilon)^{k_x} - \pi(\phi^*))z$$
$$= \varepsilon^{k_w}(1-\varepsilon)^{k_x}((1-\varepsilon)^{n-k_w-k_x} y + (1 - (1-\varepsilon)^{n-k_w-k_x})z),$$

which from our choice of ε is negative. This contradicts the fact that b is an equilibrium of $\Gamma(\to, \pi)$ and $b_i(\theta) = r$.

($\Leftarrow$) Let $\pi \in \Delta\Theta$ and let $f \in \underline{F}$, where $f(w, \ldots, w) = r$ for all $i \in N$. Show that $f \leqq BR(f)$, where $\leqq$ and BR are defined in the proof of Lemma 1. Say that $f_i(\theta) = r$. Then $\theta_i = w$ and $u_i(w, r, f_{N\setminus\{i\}}(\phi)) \geqq u_i(w, s, f_{N\setminus\{i\}}(\phi))$ for all $\phi \in P_i(\theta)$. Hence $\sum_{\phi\in P_i(\theta)} \pi(\phi)(u_i(w, r. f_{N\setminus\{i\}}(\phi)) - u_i(w, s, f_{N\setminus\{i\}}(\phi))) \geqq 0$ and thus $BR_i(f)(\theta) = r$. Thus $f \leqq BR(f)$. As in the proof of Lemma 1, therefore there exists an equilibrium b such that $f \leqq b$. Since $f_i(w, \ldots, w) = r$ for all $i \in N$, we know that $b_i(w, \ldots, w) = r$ for all $i \in N$. ||

Lemma 3. *If* $\to \subset \to'$ *and* $\to$ *is a sufficient network, then* $\to'$ *is a sufficient network.*

Proof. Say that $C \subset N$ is "big enough" for $i \in N$ if a willing i wants to revolt as long as everyone in C revolts, that is, $u_i(w, r, a_{N\setminus\{i\}}) \geqq u_i(w, s, a_{N\setminus\{i\}})$, where $a = (r_C, s_{N\setminus C})$. Obviously since utilities are supermodular, if C is big enough for i, then $D \supset C$ is big enough for i. Also, saying $f \in \underline{F}$ is equivalent to saying $f_i(\theta) = r$ if and only if $\theta_i = w$ and $\{j \in N: f_j(\phi) = r\}$ is big enough for i for all $\phi \in P_i(\theta)$.

Say F' is the set of strategy profiles given $\to'$. Similarly, say $B'(i)$ is the ball given $\to'$ and $\mathscr{P}'_i$ is person i's partition given $\to'$. Since $\to \subset \to'$, we have $B(i) \subset B'(i)$ and thus $P_i(\theta) \supset P'_i(\theta)$ for all $\theta \in \Theta$; the partition $\mathscr{P}_i$ is "coarser" than the partition $\mathscr{P}'_i$. Hence if f_i is measurable with respect to $\mathscr{P}_i$, it is also measurable with respect to $\mathscr{P}'_i$. Hence $F \subset F'$.

Define $\underline{BR}: F \to F$ as $\underline{BR}_i(f)(\theta) = r$ if $\theta_i = w$ and $\{j \in N: f_j(\phi) = r\}$ is big enough for i for all $\phi \in P_i(\theta)$ and $\underline{BR}_i(f)(\theta) = s$ otherwise. Similarly, define $\underline{BR}': f' \to f'$ as $\underline{BR}'_i(f)(\theta) = r$ if $\theta_i = w$ and $\{j \in N: f_j(\phi) = r\}$ is big enough for i for all $\phi \in P'_i(\theta)$ and $\underline{BR}'_i(f)(\theta) = s$ otherwise. Obviously $f' \in \underline{F}'$ if and only if $f' = \underline{BR}(f')$. Let $\leqq$ be defined on F and F' as in the proof of Lemma 1.

Show that for all $f \in F$, $\underline{BR}(f) \leq \underline{BR}'(f)$. Say $\underline{BR}_i(f)(\theta) = r$. Hence we have $\theta_i = w$ and $\{j \in N: f_j(\phi) = r\}$ is big enough for i for all $\phi \in P_i(\theta)$. But $P_i(\theta) \supset P'_i(\theta)$; hence $\{j \in N: f_j(\phi) = r\}$ is big enough for i for all $\phi \in P'_i(\theta)$ and thus $\underline{BR}'_i(f)(\theta) = r$.

By Lemma 2, since $\rightarrow$ is sufficient, there exists $b \in \underline{F}$ such that $b_i(w, \cdots, w) = r$ for all $i \in N$. By the definition of $\underline{BR}$ we have $b = \underline{BR}(b)$. From what we showed above, we know that $\underline{BR}(b) \leq \underline{BR}'$ and thus $b \leq \underline{BR}'(b)$. As in the proof of Lemma 1, let $E' = \{f \in F': f \leq \underline{BR}'(f)\}$ and $b' = \bigvee_{f \in E'} f$; we know that $b' = \underline{BR}'(b')$ and hence $b' \in \underline{F}'$. Since $b \geq \underline{BR}'(b)$, we know $b \in E'$ and hence $b \leq b'$. Since $b_i(w, \ldots, w) = r$ for all $i \in N$, we have $b'_i(w, \ldots, w) = r$ for all $i \in N$. Hence $\rightarrow'$ is sufficient by Lemma 2. ||

Proposition. *Say $\rightarrow$ is a minimal sufficient network. Then there exist cliques $M_1, M_2, \ldots, M_z$ which cover N and a binary relation $\rightarrow$ defined over $M_1, M_2, \ldots, M_z$ such that* (1) *$i \rightarrow j$ if and only if there exists some M_k which contains i and M_l which contains j such that $M_k \rightarrow M_l$ and* (2) *if $M_{i_{y-1}} \rightarrow M_{i_y}$, then there exists a totally ordered set $M_{i_1}, M_{i_2}, \ldots, M_{i_{y-1}}, M_{i_y}$, where M_{i_1} is maximal.*

Proof. By Lemma 2, there exists $f \in \underline{F}$ such that $f_i(w, \ldots, w) = r$ for all $i \in N$. Let $\underline{BR}: F \rightarrow F$ be defined as in the proof of Lemma 3. As in the proof of Lemma 1, let $b = \bigvee_{f \in E} f$, where $E = \{f \in F: f \leq \underline{BR}(f)\}$. We know $b = BR(b)$ and hence $b \in \underline{F}$. Since $f \in \underline{F}$, we know $f \in E$ and hence $f \leq b$. Thus $b_i(w, \ldots, w) = r$ for all $i \in N$. As in the proof of Lemma 1, it is easy to show that b is increasing.

Let $W = \{(C, i): i \in C \subset N$ and $C = \{j \in N: b_j(\theta) = r\}$ for some $\theta \in \Theta\}$. Let $X = \{(C, i) \in W: \nexists (D, i) \in W$ such that $D \subset C, D \neq C\}$. In other words, if $(C, i) \in X$, then C is a minimal set such that $i \in C$ and C is the set of people who revolt in some state of the world. We have the following facts. Fact 1: For all $i \in N$, there exists $(C, i) \in X$. This is true because $b_i(w, \ldots, w) = r$ for all $i \in N$ and hence $(N, i) \in W$ for all $i \in N$. Fact 2: If $(C, i) \in X$ and $j \in C$, then $\exists (D, j) \in X$ such that $D \subset C$. This is true because if $(C, i) \in X$ and $j \in C$, then $(C, j) \in W$. Fact 3: If $(C, i) \in X$, then C is big enough for i. This is true because $(C, i) \in X \subset W$ means that there exists $\theta \in \Theta$ such that $C = \{j \in N: b_j(\theta) = r\}$. Since $i \in C$, we have $b_i(\theta) = r$; since $b \in \underline{F}$ and $\theta \in P_i(\theta)$, we know that C must be big enough for i. Fact 4: If $(C, i) \in X$ and $j \in C$, then $j \rightarrow i$. Since $(C, i) \in X$, there exists $\theta \in \Theta$ such that $C = \{j \in N: b_j(\theta) = r\}$. Let $D = \{j \in N: b_j(\theta_{B(i)}, x_{N \setminus B(i)}) = r\}$. By measurability of b_i, we know that $i \in D$. Hence $(D, i) \in W$. Since unwilling people never revolt, we know $D \subset B(i)$. Since b is increasing, we know that $D \subset C$. Since $(C, i) \in X$, we must have $C = D$ and hence $C \subset B(i)$; in other words, if $j \in C$, then $j \rightarrow i$.

Define $\rightarrow^*$ on N: say $j \rightarrow^* i$ if $\exists (C, i) \in X$ such that $j \in C$. Show that $\rightarrow^*$ is a sufficient network. Let $B^*(i) = \{j \in N: j \rightarrow^* i\}$ and define $P^*_i(\theta)$ and $\mathscr{P}^*_i$ accordingly. Define $f^*_i: \Theta \rightarrow \{r, s\}$: say $f^*_i(\theta) = r$ if $\exists (C, i) \in X$ such that $\theta_C = (w, \ldots, w)$ and say $f^*_i(\theta) = s$ otherwise. Whenever $(C, i) \in X$ we have $C \subset B^*(i)$ by our definition of $\rightarrow^*$, and hence f^*_i is measurable with respect to $\mathscr{P}^*_i$.

Define $\underline{BR}^*: F^* \rightarrow F^*$ as $\underline{BR}^*_i(f)(\theta) = r$ if $\theta_i = w$ and $\{j \in N: f_j(\phi) = r\}$ is big enough for all $\phi \in P^*_i(\theta)$ and $\underline{BR}^*_i(f)(\theta) = s$ otherwise. Show that $f^* \leq \underline{BR}^*(f^*)$. Say $f^*_i(\theta) = r$. Hence $\exists (C, i) \in X$ such that $\theta_C = (w, \ldots, w)$. Since $\theta_i = w$, and since C is big enough for i by Fact 3, it suffices to show that for all $\phi \in P^*_i(\theta), f^*_j(\phi) = r$ for all $j \in C$. Let $\phi \in P^*_i(\theta)$. Since $C \subset B^*(i)$ and $\theta_C = (w, \ldots, w)$, we have $\phi_C = (w, \ldots, w)$. Let $j \in C$. By Fact 2 there exists $(D, j) \in X$ such that $D \subset C$, and hence $\phi_D = (w, \ldots, w)$. Hence $f^*_j(\phi) = r$ by our definition of f^* above.

As in the proof of Lemma 1, therefore there exists $f^{**} \in \underline{F}^*$ such that $f^* \leq f^{**}$. Since $f^*_i(w, \ldots, w) = r$ for all $i \in N$ by Fact 1 and the definition of f^*, we have $f^{**}_i(w, \ldots, w) = r$ for all $i \in N$, and therefore $\rightarrow^*$ is a sufficient network by Lemma 2. By Fact 4 and the definition of $\rightarrow^*$, we have $\rightarrow^* \subset \rightarrow$. Since $\rightarrow$ is a minimal sufficient network, we have $\rightarrow^* = \rightarrow$. In other words, we have Fact 5: for all $i, j \in N$, $j \rightarrow i$ if and only if $\exists (C, i) \in X$ such that $j \in C$.

Define the binary relation $>$ on X as follows: $(C, i) > (D, j)$ if $C \subset D$ and $i \rightarrow j$. If $(C, i) \in X$, since $i \in C$ we have $i \rightarrow i$; hence $>$ is reflexive. To show that $>$ is transitive, let $(C, i) > (D, j)$ and $(D, j) > (E, k)$. Since $C \subset D$ and $D \subset E$, we know that $C \subset E$. Hence $i \in E$ and since $(E, k) \in X$, we know that $i \rightarrow k$ by Fact 5. So $(C, i) > (E, k)$.

Now define the relation $< >$ on X as $(C, i) < > (D, j)$ if $(C, i) > (D, j)$ and $(D, j) > (C, i)$. Since $>$ is reflexive and transitive, so is $< >$, and obviously $< >$ is symmetric. Hence $< >$ is an equivalence relation, and thus partitions X into equivalence classes $X_1, X_2, \ldots, X_y$; that is, $(C, i) < > (D, j)$ if and only if $(C, i), (D, j) \in X_k$.

Define $X_k > X_l$ if $(C, i) > (D, j)$ for all $(C, i) \in X_k$ and $(D, j) \in X_l$. Since $>$ is transitive, it is easy to show that if $(C, i) \in X_k$, $(D, j) \in X_l$, and $(C, i) > (D, j)$, then $X_k > X_l$. It is easy to show that $>$ as defined over equivalence classes is transitive and acyclic.

Now define $\mu(X_k) = \{i \in N: \exists C \subset N$ such that $(C, i) \in X_k\}$. Let $\{M_1, \ldots, M_z\} = \{\mu(X_1), \mu(X_2), \ldots, \mu(X_y)\}$. Because there exists $(C, i) \in X$ for all $i \in N$ by Fact 1, we know that $M_1, \ldots, M_z$ cover N. Let $i, j \in M_k$. There exists $X_{k'}$ such that $M_k = \mu(X_{k'})$. Hence there exists (C, i), $(D, j) \in X_{k'}$; thus $(C, i) > (D, j)$, and hence $i \rightarrow j$. So each M_k is a clique. Finally, define $M_k \rightarrow M_l$ if there exists $X_{k'}, X_{l'}$ such that $M_k = \mu(X_{k'}), M_l = \mu(X_{l'})$, and $X_{k'} > X_{l'}$.

To show (1), first say $i \rightarrow j$. Hence by Fact 5 there must exist $(D, j) \in X$ such that $i \in D$. By Fact 2, $\exists (C, i) \in X$ such that $C \subset D$. Hence $(C, i) > (D, j)$. Say $(C, i) \in X_{k'}$ and $(D, j) \in X_{l'}$. Hence $X_{k'} > X_{l'}$ and thus $M_k \rightarrow M_l$, where

$M_k = \mu(X_{k'})$ and $M_l = \mu(X_{l'})$, and obviously $i \in M_k$ and $j \in M_l$. To show the other direction, let $M_k \rightarrow M_l$. We thus have $X_{k'}, X_{l'}$ such that $M_k = \mu(X_{k'}), M_l = \mu(X_{l'})$, and $X_{k'} > X_{l'}$. If $i \in M_k$ and $j \in M_l$, there exists $(C, i) \in X_{k'}$ and $(D, j) \in X_{l'}$, and since $X_{k'} > X_{l'}$, we have $(C, i) > (D, j)$ and so $i \rightarrow j$.

To show (2), say we have $M_{i_{y-1}} \rightarrow M_{i_y}$. Hence there exist $X_{i'_{y-1}}, X_{i'_y}$ such that $M_{i_{y-1}} = \mu(_{i'_{y-1}})$, $M_{i_y} = \mu(X_{i'_y})$, and $X_{i'_{y-1}} > X_{i'_y}$. Since $>$ over equivalence classes is transitive and acyclic, there exists a totally ordered set $X_{i'_1}, \ldots, X_{i'_{y-1}}, X_{i'_y}$, where $X_{i'_1}$ is maximal. Hence $\mu(X_{i'_1}), \ldots, \mu(X_{i'_{y-1}})$, $\mu(X_{i'_y})$ is totally ordered by $\rightarrow$ and $\mu(X_{i'_1})$ is maximal with respect to $\rightarrow$. ||

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