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Minority Voting Rights Can Maximize Majority Welfare

MICHAEL SUK-YOUNG CHWE *University of Chicago*

I use Condorcet's information aggregation model to show that sometimes the best possible decision procedure for the majority *allows the minority to "enforce" its favored outcome even when overruled by a majority. "Special" voting power gives the minority an incentive to participate meaningfully, and more participation means more information is aggregated, which makes the majority better off. This result can be understood as a mathematical corroboration of Lani Guinier's arguments that voting procedures can be designed to encourage minority participation, benefitting everyone.*

The most immediate criterion of democratic decision making is fairness. But Condorcet (1785) in his "jury theorem" argued on welfarist grounds, showing that if each individual has an equal chance of having the correct opinion, then majority rule is most likely to select the better of two alternatives. Minority representation is almost always discussed in terms of fairness. I use Condorcet's information aggregation argument to show that sometimes the best possible decision procedure for the majority allows the minority to "enforce" its favored outcome even when overruled by a majority. This is simply because of the importance of participation: "Special" voting power gives the minority an incentive to participate meaningfully, and more participation means more information is aggregated, which makes the majority better off. That majority rule can discourage minority participation, and that voting procedures can be designed to encourage it, benefitting everyone, are points made by Lani Guinier (1994); this article might be considered a mathematical corroboration. Proponents of minority voting rights and procedures other than majority rule need not concede welfare, efficiency, or the vision, however appealing or unappealing, of a world in which we have different opinions but deep down all want the same thing.

The idea that procedures should be designed to prevent a majority from dominating a minority is often traced to James Madison and John Stuart Mill. But neither emphasized minority participation as valuable in itself. Madison (1788) saw minority participation mainly as preventing the "mischiefs of faction": "Extend the sphere and you take in a greater variety of parties and interests; you make it less probable that a majority of the whole will have a common motive to invade the rights of other citizens." Mill valued minority participation mainly for political education. For example, a transferable vote system would allow the election of "hundreds of able men of independent thought, who would have no chance whatever of being chosen by the majority of any existing constituency.... [I]f the presence in the representative assembly can be insured of even a few of the first minds in the country, though the remainder consist only of average minds, the influence of these leading spirits is sure to make itself sensibly felt" (Mill 1861, chap. 7; see also Thompson 1976, 69–70). The simple idea that everyone's opinion is equally worthwhile, and hence minority participation should be encouraged, differentiates my argument from that of Mill and Madison. In my model, the majority prefers minority participation for the very direct and "selfish" reason of having the minority's information included.

Guinier argues for the general consideration of procedures other than majority rule, such as cumulative voting (see also Brischetto 1995; Still and Karlan 1995). This article uses game theory's powerful "mechanism design" approach to find the best among all possible procedures, as opposed to the more common practice of predicting behavior given a particular procedure. After presenting the model and the main results, I go through the simplest example, in which there are just three voters, and explain the results intuitively. I then consider "anonymous" procedures, which depend only on vote totals and not people's identities, and obtain similar results. Finally, I consider possible objections (for example, the results here apply better to small rather than large groups) and conclude by arguing that Condorcet's jury theorem is much more than just an argument for majority rule; the idea of information aggregation allows each person to benefit from the inclusion and participation of others.

THE MODEL

I use a "drug research" analogy to explain the model (for surveys of the information aggregation literature, see Grofman and Owen 1986; Grofman and Withers 1993; Miller 1986; Young 1995). A group of n people, where n is odd and at least 3, have to choose whether to legalize drug a or drug b, and not both. One of them works with a greater probability, but no one knows which one. To find out which drug is truly superior, each person takes the drugs herself and sees which works better: Each person's private evidence, which is not completely conclusive, either favors a or b. Each person then makes a report to a central computer,

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which follows a predetermined procedure and outputs the group's decision.

In the political context, choosing a drug would be choosing between two candidates for office, whether or not to build a nuclear power plant, whether or not to intervene in a foreign war, and so on. A person's private evidence would be his independent information about the issue. A person's report would be her vote. The computer program is the decision procedure. Under majority rule, for example, the computer would simply choose the drug that receives more votes.

So far this is Condorcet's model. I add two things. First, people need not be identical; they may have different prior beliefs about which drug is superior or different preferences over the drugs (one drug may be fine for you but have unpleasant side effects for me, regardless of how well it works). A person's idiosyncracies may influence what he chooses to report; someone with a strong prior belief or preference toward a may report a even if his private experiment supports b. Second, people act strategically (Austen-Smith and Banks 1996; Feddersen and Pesendorfer 1996): when deciding what to report, a person considers how it affects the group's decision, necessarily taking into account the decision procedure and other people's strategies.

Mathematically speaking, each person $i \in N = \{1, 2, \ldots, n\}$ has a prior belief that a is superior with probability $\pi_i(a)$ and b is superior with probability $\pi_i(b)$, where $\pi_i(a), \pi_i(b) \in [0, 1]$ and $\pi_i(a) + \pi_i(b) = 1$. Say that each person's private evidence is correct with probability q, where $q \in (1/2, 1)$; the closer q is to 1, the stronger is the evidence. We say that $g(d, e)$ is the probability that a person's private evidence supports e when the superior drug truly is d; thus we have $g(d, e) = q$ if $d = e$ and $g(d, e) = 1 - q$ if $d \neq e$.

The decision procedure is a function $f: \{a, b\}^n \times \{a, b\} \to [0, 1]$, where $f(r_1, \ldots, r_n, a)$ is the probability of choosing a and $f(r_1, \ldots, r_n, b)$ is the probability of choosing b, given the reports $r_1, \ldots, r_n \in \{a, b\}$. Of course $f(r_1, \ldots, r_n, a), f(r_1, \ldots, r_n, b) \in [0, 1]$, and $f(r_1, \ldots, r_n, a) + f(r_1, \ldots, r_n, b) = 1$. After the decision is made, each person gets utility $u_i(d, c)$ when the superior drug truly is d and the drug chosen is c. We assume that $u_i(a, a) > u_i(a, b)$ and $u_i(b, b) > u_i(b, a)$; in other words, everyone prefers the superior drug.

To summarize, the model is characterized by the number of people n, prior beliefs π_i, utility functions u_i, and the strength of evidence q, all "exogenous parameters." The "policy variable" is f, the decision procedure.

Each person's strategy is a choice of what to report given his evidence. Hence person i's strategy is a function $s_i: \{a, b\} \to \{a, b\}$. For example, the identity function id, where $id(a) = a$ and $id(b) = b$, is the strategy in which a person always reports his result truthfully. Let S be the set of all strategies, which is the same for everyone.

Given strategies $s_1, \ldots, s_n$, the probability that the group chooses drug c, given that the superior drug truly is d, is

$$p(s_1, \ldots, s_n, d, c) = \sum_{(e_1, \ldots, e_n) \in \{a,b\}^n} g(d, e_1) \cdots g(d, e_n) f(s_1(e_1), \ldots, s_n(e_n), c).$$

Here $e_1, \ldots, e_n$ is the evidence, $s_1(e_1), \ldots, s_n(e_n)$ are the reports given this evidence, and $f(s_1(e_1), \ldots, s_n(e_n), c)$ is the probability of choosing c given the reports. Hence, given strategies $s_1, \ldots, s_n$, and the decision procedure f, person i's expected utility is $EU_i(f, s_1, \ldots, s_n) = \pi_i(a)p(s_1, \ldots, s_n, a, a)u_i(a, a) + \pi_i(a)p(s_1, \ldots, s_n, a, b)u_i(a, b) + \pi_i(b)p(s_1, \ldots, s_n, b, a)u_i(b, a) + \pi_i(b)p(s_1, \ldots, s_n, b, b)u_i(b, b)$.

Equilibrium is defined in the usual way: an n-tuple of strategies from which no one can gain by deviating to another strategy. We say that $(s_1, \ldots, s_n) \in S^n$ is an *equilibrium* if $EU_i(f, s_1, \ldots, s_n) \geq EU_i(f, s_1, \ldots, s_{i-1}, s_i', s_{i+1}, \ldots, s_n)$ for all $s_i' \in S$ and for all $i \in N$. Given a procedure f, there might be many equilibria; if $(id, \ldots, id)$, everyone telling the truth, is an equilibrium, we assume it is "focal" (Schelling 1980) and hence will be the one actually played. If $(id, \ldots, id)$ is an equilibrium, we say that the procedure f is *incentive compatible*.

Since $p(s_1, \ldots, s_n, a, b) = 1 - p(s_1, \ldots, s_n, a, a)$ and $p(s_1, \ldots, s_n, b, a) = 1 - p(s_1, \ldots, s_n, b, b)$, we have $EU_i(f, s_1, \ldots, s_n) = p(s_1, \ldots, s_n, a, a)\pi_i(a)(u_i(a, a) - u_i(a, b)) + p(s_1, \ldots, s_n, b, b)\pi_i(b)(u_i(b, b) - u_i(b, a)) + \pi_i(a)u_i(a, b) + \pi_i(b)u_i(b, a)$. The last two terms here are constant in $s_1, \ldots, s_n$. It is also apparent that all specifications of π_i and u_i that keep the ratio of $\pi_i(a)(u_i(a, a) - u_i(a, b))$ to $\pi_i(b)(u_i(b, b) - u_i(b, a))$ the same are equivalent in terms of person i's decision. Since both of these terms are nonnegative by assumption, without loss of generality we can assume that $u_i(a, a) = 1$, $u_i(a, b) = 0$, $u_i(b, a) = 0$, $u_i(b, b) = 1$.

In other words, all relevant characteristics of person i can be represented by π_i alone. For example, if $\pi_i(a) > \pi_i(b)$, this can be understood either as person i having a high prior belief that the correct drug is a or person i simply having a strong preference for a; when $\pi_i(a) > \pi_i(b)$, we simply say that person i is "biased toward" a (see also Calvert 1985; the term "bias" is associated with prejudice or partiality, but here, of course, I use it as shorthand description, not judgment). For various reasons, some theorists prefer to assume that people always have the same prior beliefs (Aumann 1976; Harsanyi 1968; but see also Aumann 1987; Morris 1995); in my model different prior beliefs can simply be a way of representing the more typical assumption of identical beliefs but differing preferences. For example, the finding that in the United States 63% of blacks but only 33% of whites support increased spending on government services (Dawson 1994, 183) can be understood as a difference either in preference or in predisposition.

OPTIMAL INCENTIVE COMPATIBLE DECISION PROCEDURES

To find a good decision procedure, we naturally should consider which equilibrium strategies result from a particular procedure. But it turns out that we only need consider procedures in which everyone telling the truth is an equilibrium; that is, procedures that are incentive compatible. This is because of the "revelation principle" (Myerson 1991), which says that any equilibrium for any procedure f can be represented as equilibrium truth-telling in another procedure f'. Thus, if we look at incentive compatible procedures, we are actually looking at all equilibria for all possible procedures. We say that f is an *optimal incentive compatible procedure* for person i if f maximizes $EU_i(f, id, \ldots, id)$ subject to the constraint that f is incentive compatible.

How do we find an optimal incentive compatible procedure? For $(id, \ldots, id)$, everyone telling the truth, to be an equilibrium, it must be that no one can gain by deviating. Say that person i plays strategy id. It is easy to show that if she gains by deviating to some other strategy (including possibly a "mixed" strategy), then she must gain by either deviating to s_{aa} or s_{bb}, where s_{aa} is reporting a all the time ($s_{aa}(a) = s_{aa}(b) = a$) and s_{bb} is reporting b all the time ($s_{bb}(a) = s_{bb}(b) = b$). Hence, $(id, \ldots, id)$ is an equilibrium if and only if each person's "incentive compatibility constraints," $EU_i(f, id, \ldots, id) \geq EU_i(f, id, \ldots, s_{aa}, \ldots, id)$ and $EU_i(f, id, \ldots, id) \geq EU_i(f, id, \ldots, s_{bb}, \ldots, id)$, are satisfied.

Remember that $f(r_1, \ldots, r_n, a)$ gives the probability of choosing a, and $f(r_1, \ldots, r_n, b)$ gives the probability of choosing b, given the reports $r_1, \ldots, r_n \in \{a, b\}$. So we can think of f simply as 2^{n+1} nonnegative numbers subject to the constraints $f(r_1, \ldots, r_n, a) + f(r_1, \ldots, r_n, b) = 1$. Note that expected utilities are linear in these numbers, and hence the incentive compatibility constraints are simply linear inequalities. Thus, finding an optimal incentive compatible procedure is a linear programming problem.

RESULTS

When everyone is relatively unbiased, we get Condorcet's jury theorem: The optimal incentive compatible procedure is majority rule.

RESULT 1. *Say* $\pi_1(a), \ldots, \pi_n(a) \in (1 - q, q)$. *Then the unique optimal incentive compatible procedure for person* i *is majority rule* f_{mr}, *defined by*

$$f_{mr}(r, a) = \begin{cases} 1 & \text{if } \#\{i \in N : r_i = a\} \geq (n + 1)/2 \\ 0 & \text{otherwise} \end{cases}$$

where $f_{mr}(r_1, \ldots, r_n, b) = 1 - f_{mr}(r_1, \ldots, r_n, a)$.

Actually, when everyone is relatively unbiased, incentive compatibility is not an issue: If a procedure is optimal for person i, since people have very similar preferences, it is optimal for everyone, and hence no one can gain by deviating (see McLennan 1998 more generally).

Say now that persons 1 to m are a minority, and persons $m + 1$ to n are a majority, where $m \leq (n - 1)/2$. When everyone is not too biased, when $\pi_1(a), \ldots, \pi_n(a) \in (1 - q, q)$, majority rule is the best incentive compatible procedure. Say that minority beliefs become slightly more biased, toward a for example, so that $\pi_1(a) = \cdots = \pi_m(a)$ becomes slightly greater than q. Then the "minority caucus" procedure is an optimal incentive compatible procedure for a person in the majority.

RESULT 2. *Let* $M = \{1, 2, \ldots, m\}$ *be the minority, where* $m \leq (n - 1)/2$. *Say that majority beliefs* $\pi_{m+1}(a), \ldots, \pi_n(a) \in (1 - q, q)$. *Then there exists* $\delta > 0$ *such that when minority beliefs* $\pi_1(a) = \ldots = \pi_m(a) \in (q, q + \delta)$, *the minority caucus procedure* f_{mc} *is an optimal incentive compatible procedure for a person in the majority, where* f_{mc} *is defined by*

$$f_{mc}(r, a) = \begin{cases} 1 & \text{if } \#\{i \in N : r_i = a\} \geq (n + 1)/2 \\ 1 & \text{if } \#\{i \in N : r_i = a\} = (n - 1)/2 \text{ and } \#\{i \in M : r_i = a\} \geq k + 1 \\ z & \text{if } \#\{i \in N : r_i = a\} = (n - 1)/2 \text{ and } \#\{i \in M : r_i = a\} = k \\ 0 & \text{otherwise} \end{cases}$$

where $k \in \{0, 1, \ldots, m\}$, $z \in [0, 1]$ (*if* $k = m$, *then* $z > 0$), *and* $f_{mc}(r, b) = 1 - f_{mc}(r, a)$.

In the minority caucus procedure f_{mc}, first everyone votes. If a, the minority's favored candidate, wins a majority of votes, it is chosen; if it wins two votes less than a majority, then b is chosen. If a wins one vote less than a majority, however, then the minority votes alone are tallied. If the number of minority a votes is greater than some threshold k, then a is chosen; if the number of minority a votes is equal to k, then a is chosen with some probability z. So the minority can sometimes "enforce" a all by itself, even when b receives a majority of the votes. (The proof, a direct application of the Kuhn-Tucker theorem on linear programming, is in the Appendix; if the minority were biased in favor of b instead, we would, of course, have the analogous result.)

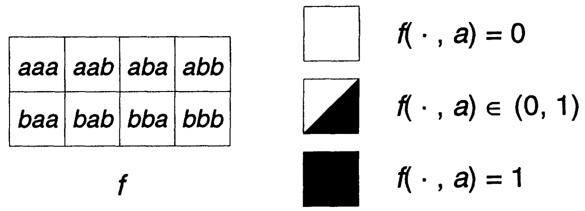
Result 2 does not claim that the minority caucus procedure is the unique optimum. But we do have the following.

RESULT 3. *Let* $M = \{1, 2, \ldots, m\}$ *be the minority, where* $m \leq (n - 1)/2$. *Say that majority beliefs* $\pi_{m+1}(a), \ldots, \pi_n(a) \in (1 - q, q)$. *Then there exists* $\delta > 0$ *such that when minority beliefs* $\pi_1(a) = \cdots = \pi_m(a) \in (q, q + \delta)$, *for any optimal incentive compatible procedure* f, *we have:*

(*i*) $f(r, a) = 1$ *for all* r *such that* $\#\{i \in N : r_i = a\} \geq (n + 1)/2$;
(*ii*) $f(r, a) = 0$ *for all* r *such that* $\#\{i \in N : r_i = a\} \leq (n - 3)/2$;
(*iii*) $f(r, a) > 0$ *for some* r *such that* $\#\{i \in N : r_i = a\} = (n - 1)/2$.

In other words, optimal incentive compatible procedures for the majority differ only in what happens in the case that a wins one vote short of a majority. In this

FIGURE 1. Representing a Procedure f Graphically When $n = 3$



case, it is always possible for the minority's favored candidate to be chosen with some probability; in other words, all optimal incentive compatible procedures for the majority involve special minority power.

THE THREE-PERSON EXAMPLE

When $n = 3$, a procedure can be represented graphically as in Figure 1. Each triple of reports (r_1, r_2, r_3) is represented as a box. Given this triple, if drug a is chosen with probability zero, then the box is left blank. If drug a is chosen with some probability greater than 0 but less than 1, then the box is partially filled in. If drug a is chosen with probability one, then the box is fully filled in.

Let the strength of evidence be $q = 2/3$. Say everyone has the same beliefs. The optimal incentive compatible procedure is shown in Figure 2. In the middle "unbiased" region, the group will simply choose the one that receives more support from the three experiments, that is, majority rule. With a little bias, the group will choose according to the bias unless all three people find evidence otherwise, which results in a unanimous vote against. When very biased, the evidence never outweighs the bias, and hence all information is ignored (see also Nitzan and Paroush 1985). One should note that this diagram shows the optimal procedure for certain intervals but not at the "endpoints" of the intervals, for example, when $\pi(a) = 1/3$. At these points, there are many optimal procedures because, conditional on some evidence, people are exactly indifferent between choosing a or b.

Now let person 1 be a minority whose bias π_1 differs from the majority $\pi_2 = \pi_3$. Now incentive compatibility matters. Figure 3 shows the optimal incentive compatible procedure for persons 2 and 3, the majority, when $\pi_2(a) = \pi_3(a) = 1/2$. Again, in the middle "unbiased" region, majority rule is incentive compatible and is best for the majority (and, as it happens, the minority). When the minority's bias $\pi_1(a)$ crosses over $q = 2/3$, however, the best incentive compatible procedure is the minority caucus procedure: Person 1, the minority, is able to get his bias with some nonzero probability when outvoted by the majority. When $\pi_1(a)$ crosses over 16/21, the best incentive compatible procedure gives person 1 the power to ensure a unilaterally, except for the case in which everyone else votes for a. But when the minority's bias $\pi_1(a)$ is greater than 4/5, he is so biased that the best incentive compatible procedures for the majority shut him out entirely, and the decision never depends on person 1's report. Since person 1's report is thrown away, it is possible for a and

FIGURE 2. The Optimal Incentive Compatible Procedure When $\pi = \pi_1 = \pi_2 = \pi_3$

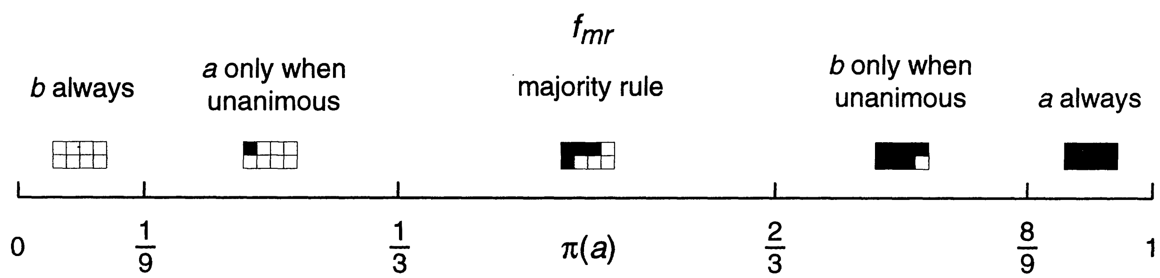


FIGURE 3. The Optimal Incentive Compatible Procedure for Persons 2 and 3, the Majority, When $\pi_2(a) = \pi_3(a) = 1/2$

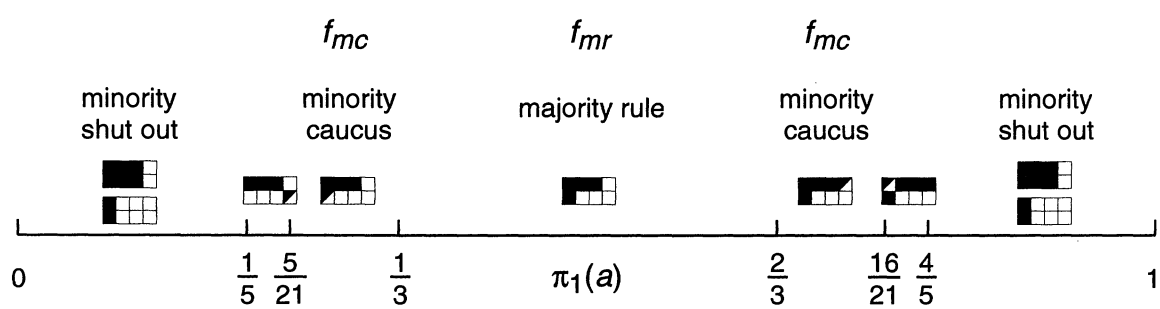
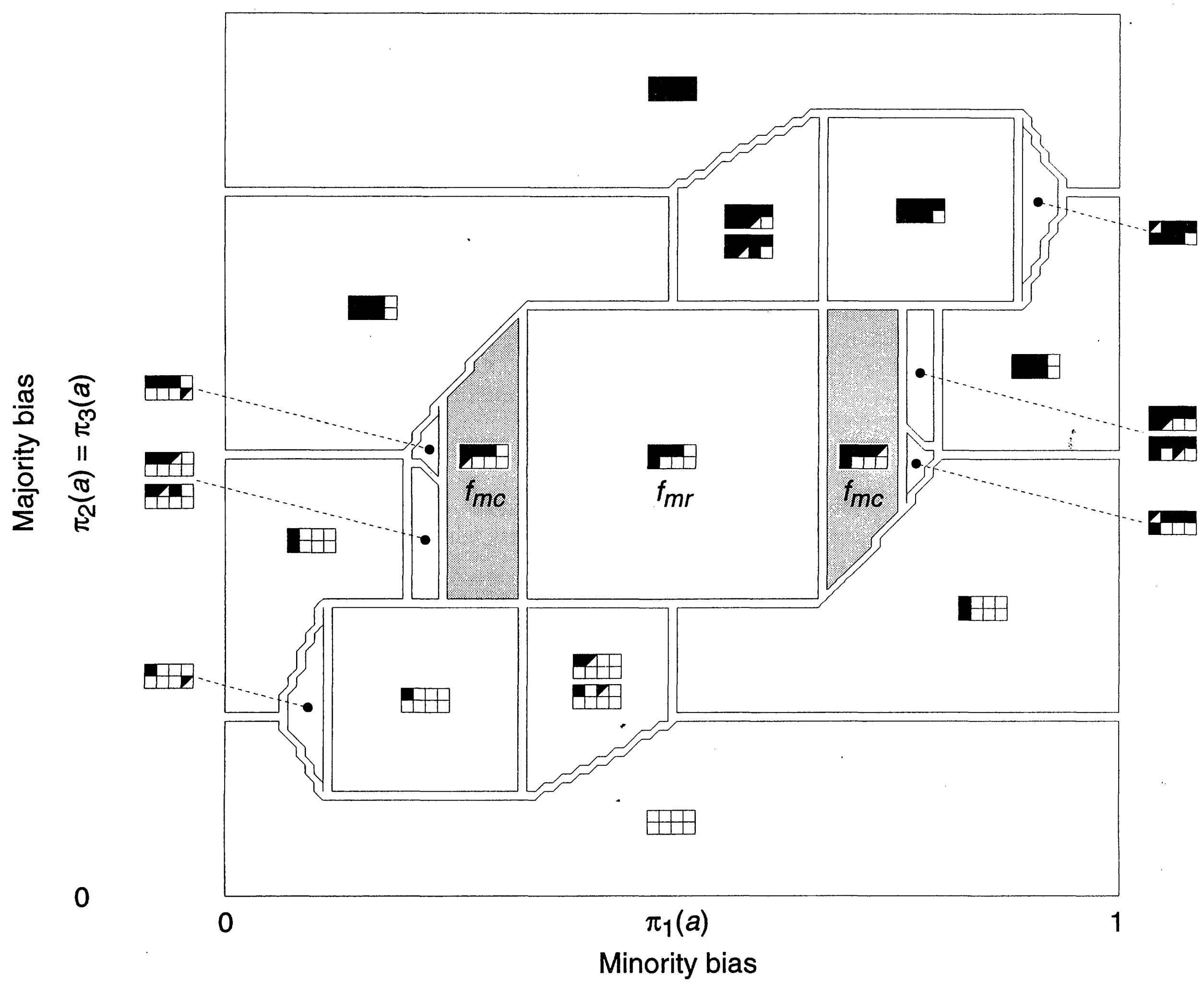


FIGURE 4. The Optimal Incentive Compatible Decision Procedure for Persons 2 and 3, the Majority

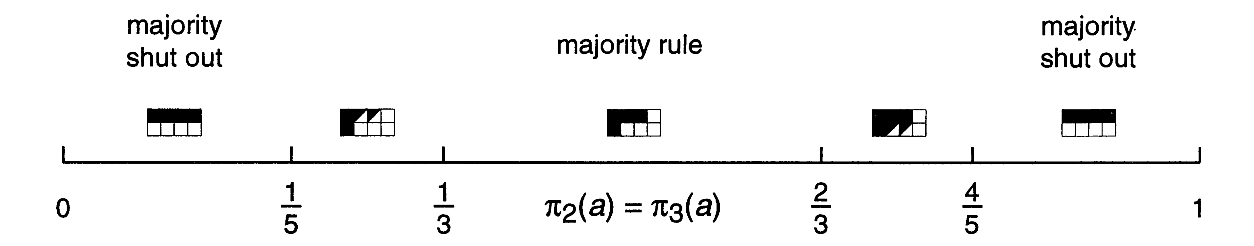


b to tie, and since the majority's prior beliefs are exactly "fifty-fifty," the majority is indifferent when there is a tie. Hence, the two procedures shown, as well as probabilistic combinations of them, are all optima.

More generally, Figure 4 shows the majority's optimal incentive compatible procedure for $\pi_1(a)$, $\pi_2(a)$, $\pi_3(a) \in \{0, 1/101, 2/101, \ldots, 100/101, 1\}$, where $\pi_2(a) = \pi_3(a)$ (a computer program is available from the author). A horizontal "slice" at $\pi_2(a) = \pi_3(a) = 1/2$ corresponds to Figure 3, and a diagonal "slice" where $\pi_1(a) = \pi_2(a) = \pi_3(a)$ corresponds to Figure 2. Again, because this is a linear programming problem, the set of optima is always a convex set. Usually this set is a single point, but sometimes it is a line segment, an "edge" of the feasible set. When this is the case, as in Figure 3, I show the procedures that are the "endpoints" of this line segment. Note that when the majority is very biased, it prefers to ignore the evidence and choose according to its bias all the time. When the minority is very biased, or even when it is only slightly biased but the majority is biased in the opposite direction, the best procedure for the majority shuts the minority out.

Why is the minority caucus procedure optimal? Say we are in the right-hand shaded region of Figure 4, where person 1 is biased toward a. Here he is biased enough that he wants the group's decision to be a even if two of the three experiments support b, but not so biased that he wants a unconditionally; if all three experiments support b, he wants the group's decision to be b. In this region, the majority is relatively unbiased. The majority would like to have majority rule, but majority rule is not incentive compatible because of person 1's bias. Say that person 1's evidence supports b. Under majority rule, person 1 never has a reason to vote for b: If the other two votes are both a, person 1's vote does not matter, since a already has majority support. If the other votes are split between a and b, then person 1 votes a since he wants the choice to be a. If the other votes are both b, person 1's vote again does not matter. Hence, for person 1, voting for a has either no effect or a good effect. Voting for a is

FIGURE 5. The Optimal Incentive Compatible Procedure for Person 1, the Minority, When $\pi_1(a)$ = 1/2 and $\pi_2 = \pi_3$



never bad. Therefore, person 1 will vote for *a* when his evidence supports *b*, and thus majority rule is not incentive compatible (Austen-Smith and Banks 1996, 34).

The minority caucus procedure chooses *a* with some probability when person 1 is the only one who votes for it. This gives person 1, the minority, the power sometimes to overrule the majority in favor of his own bias. But it also gives person 1 the incentive to report the truth. To see why, say again that person 1's evidence supports *b*. How will he vote? We saw above that under majority rule, there is no circumstance in which voting for *a* is bad. But under the minority caucus procedure, there is. If everyone else's evidence also supports *b*, then person 1 wants the group's decision to be *b*. If person 1 votes for *a*, because of his special power, then there is a chance that the group will make the "mistake" of choosing *a* instead. This is sufficient reason to make person 1 vote for *b* when his evidence supports *b*. When his evidence supports *a*, person 1 will clearly vote for *a*, and hence the minority caucus procedure is incentive compatible.

Roughly speaking, in the shaded regions of Figure 4, the majority ideally prefers majority rule, but majority rule is not incentive compatible: The minority will always report his bias. The majority can avoid this problem with a procedure that shuts the minority out, but then the minority's evidence would be thrown away; it would not enter into the group's decision. Instead of excluding the minority, majority rule could be modified in favor of the minority's bias. This modification has two effects. Because the procedure is now biased, the majority becomes worse off. But if the modification is incentive compatible, the minority's evidence is now included in the group's decision, and this makes the majority better off. The minority caucus procedure, a modification of majority rule, is optimal for the majority when the positive effect outweighs the negative.

Very roughly speaking, with power comes discretion. When the minority has the power to impose her bias, she also has the power to impose it exactly when she does not want to, when everyone else's evidence is against it.

For comparison's sake we can look at the minority's optimal procedure as the majority's bias changes, shown in Figure 5. Here $\pi_1(a) = 1/2$ throughout. Again, when the majority is not too biased, then majority rule is optimal. When the majority becomes slightly biased toward *a*, when $\pi_2(a) = \pi_3(a)$ crosses over 2/3, the best procedure for the minority allows each majority member to have extra power. Each majority member can enforce *a* with some probability even when the other two votes are for *b*. When the majority is very biased, the best procedure for the minority is to shut out the majority completely, and the minority simply decides on her own. The general pattern of Figure 5 is thus quite similar to the pattern in Figure 3. This suggests that it does not matter much whether a majority or minority is biased; what is important is that each person values the participation of others, and that participation can be encouraged by giving people extra power.

ANONYMOUS PROCEDURES

Explicitly giving the minority extra power might be difficult legally, politically, or practically, even though it is in the majority's interest. Thus we might limit ourselves to "anonymous" procedures that depend only on vote totals and not people's identities. Formally, we say that a procedure *f* is *anonymous* if $f(r, a) = f(r', a)$ for all r, r' such that $\#\{i \in N : r_i = a\} = \#\{i \in N : r'_i = a\}$. It turns out that even in this restricted domain, we get a similar result. When the majority is relatively unbiased and the minority is slightly biased, the best procedure for the majority allows the minority's favored candidate to be elected with some probability even when it receives less than half the votes.

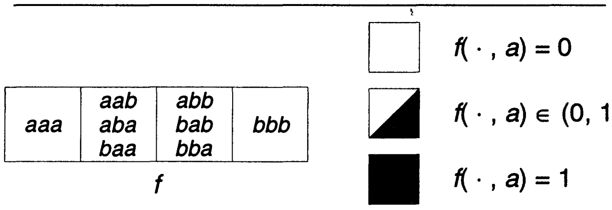
Result 4. *Let* $M = \{1, 2, \ldots, m\}$ *be the minority. Say that majority beliefs* $\pi_{m+1}(a), \ldots, \pi_n(a) \in [1/2, q)$. *Then there exists* $\delta > 0$ *such that when minority beliefs* $\pi_1(a), \ldots, \pi_m(a) \in (q, q + \delta)$, *the procedure* f_{an} *is the unique optimal anonymous incentive compatible procedure for a person in the majority, where* f_{an} *is defined by*

$$f_{an}(r, a) = \begin{cases} 1 & \textit{if } \#\{i \in N : r_i = a\} \geq (n+1)/2 \\ z & \textit{if } \#\{i \in N : r_i = a\} = (n-1)/2 \\ 0 & \textit{otherwise} \end{cases}$$

where $z \in (0, 1)$ *and* $f_{an}(r, b) = 1 - f_{an}(r, a)$.

This result differs from Result 2 in that here majority beliefs must be restricted to the interval $[1/2, q)$ as

FIGURE 6. Representing an Anonymous Procedure f Graphically When n = 3



opposed to $(1 - q, q)$. Also, here we have uniqueness and do not need the earlier simplifying assumption that everyone in the minority has the same bias. Mathematically, restricting ourselves only to anonymous procedures is just a matter of adding additional linear constraints to the linear programming problem.

When n = 3 an anonymous procedure can be notated graphically, as in Figure 6. As before, let q = 2/3 and let person 1 be a minority and persons 2 and 3 be a majority. Figure 7 shows the majority's optimal anonymous incentive compatible procedure for $\pi_1(a)$, $\pi_2(a)$, $\pi_3(a) \in \{0, 1/101, 2/101, \ldots, 100/101, 1\}$, where $\pi_2(a) = \pi_3(a)$. Interestingly, the procedure f_{an} is optimal not only when the minority is slightly biased but also when the majority is slightly biased. Figure 6 is not as complicated as Figure 4 because here we restrict ourselves to anonymous procedures; for example, the majority cannot shut the minority out because that would not be an anonymous procedure.

POSSIBLE OBJECTIONS

Here I try to anticipate three minor and three major objections. One might first object that the results are an improbable special case. The result does depend, of

FIGURE 7. The Optimal Incentive Compatible Decision Procedure for Persons 2 and 3, the Majority

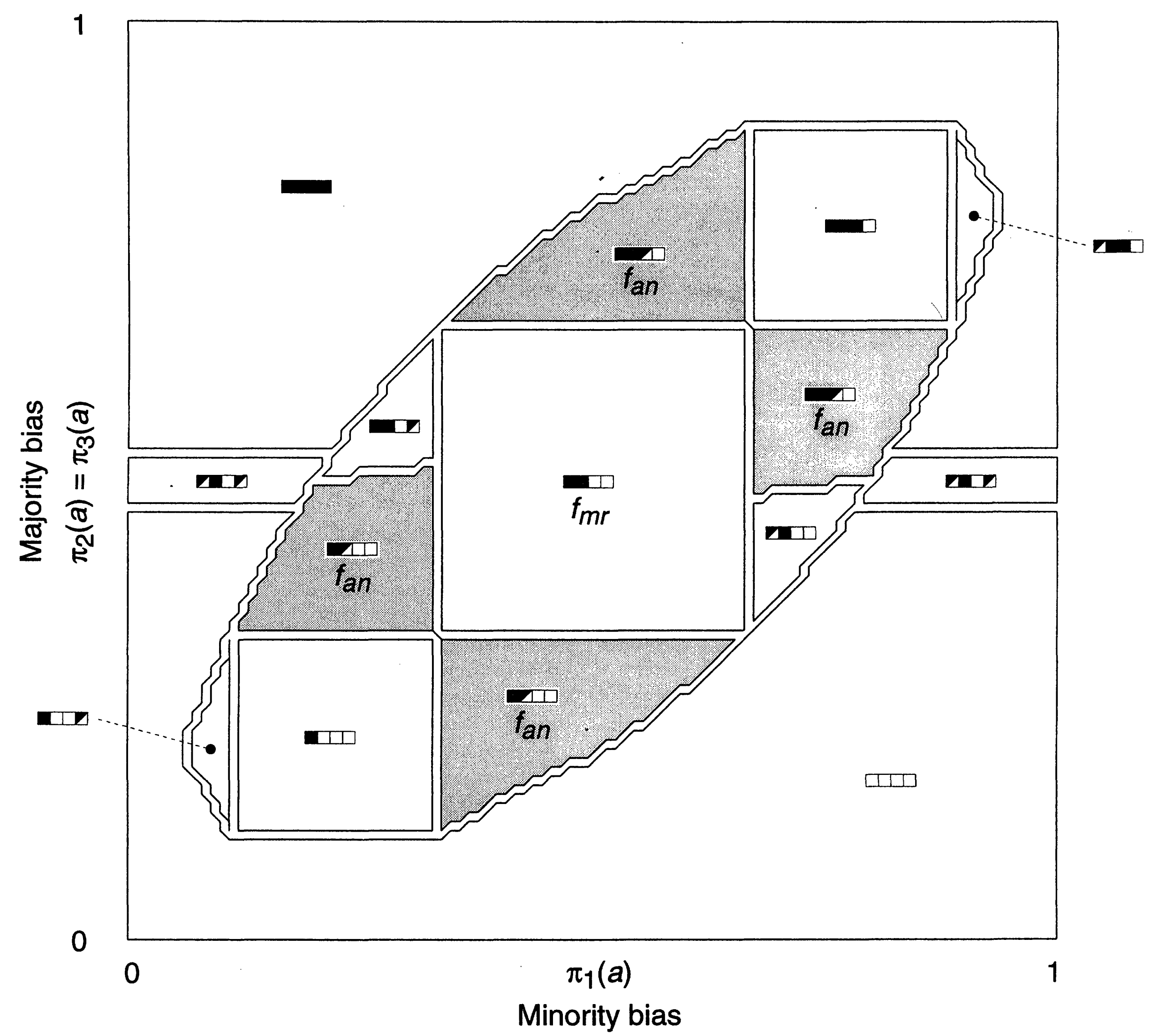
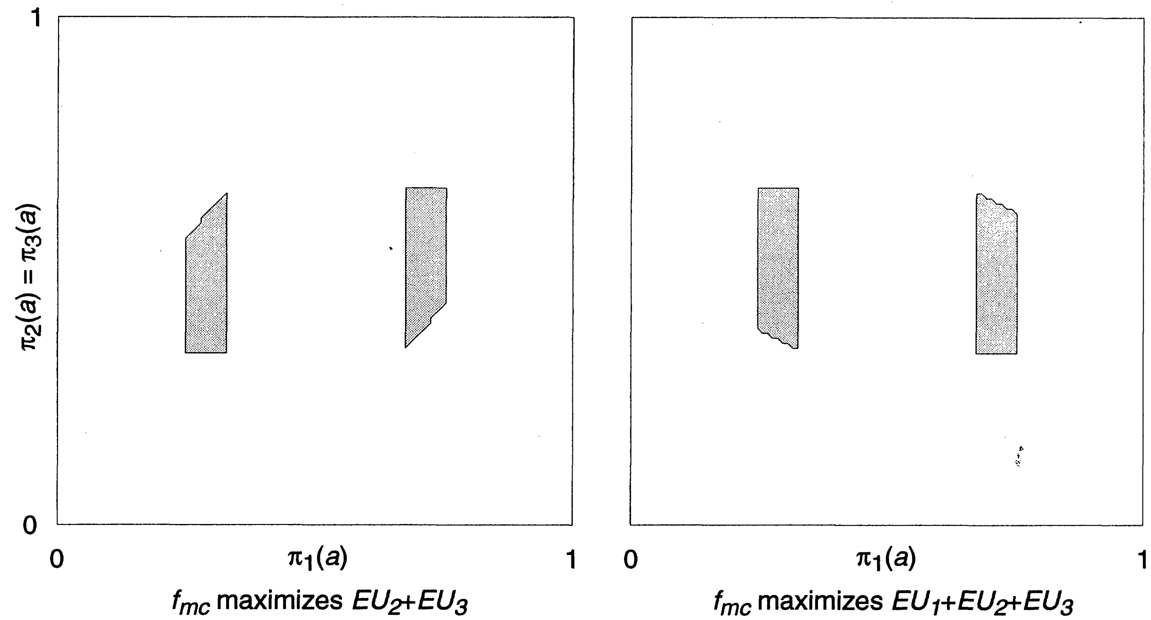


FIGURE 8. Minority Caucus Regions When the Majority's Utility Is Maximized (Left) and When the Sum of Everyone's Utility Is Maximized (Right)



course, on the parameters of the model but not in a fragile way. The "special minority power" shaded regions in figures 4 and 7, for example, are "large," comparable to the majority rule region. The welfare criterion of maximizing majority utility only is also literally extreme, but the point here is that special minority power can be optimal without giving any weight at all to minority welfare. Under different welfare criteria, results are not dramatically different. When the criterion is the sum of the three persons' utilities, for example, the regions in which the minority caucus procedure is optimal change only slightly, as shown in Figure 8.

Another objection might be that the results are less interesting as the number of voters becomes large. The minority caucus procedure differs from majority rule only when the minority's favored candidate receives just one vote short of a majority, which happens with very small probability as the number of voters increases. The simplest way to respond is to say that minority representation is important in small decision-making bodies, such as city councils, so regardless of the model's applicability to large elections, we might settle for its relevance for small groups. Another response is that gains from greater participation are essential for our results (in a similar spirit, Schotter and Weigelt [1992] show how affirmative action can enhance market efficiency by encouraging participation and hence competition). These gains are not well captured in asymptotic models. As the number of voters becomes large, one can get close to perfect information aggregation even after discarding a large fraction (almost all) of the votes (Feddersen and Pesendorfer 1997). Still another response is that it is true that the minority caucus procedure is only slightly different from majority rule, but this is because here we consider biases only slightly different from those justifying majority rule. In the "general" case, there would be a broad distribution of biases (some people slightly biased, others very biased) and multiple dimensions of choice and information. In the context of U.S. racial politics, for example, the black/white minority/majority supposed dichotomy has given way to a multiplicity including at the very least Asian, Latino, and Native Americans as well as African and European Americans. Determining optimal procedures in these more general contexts is a question for future research. The results here are just a first step. When we "perturb" Condorcet's model only slightly away from the parameter values (that is, biases) that justify majority rule, optimal procedures for the majority give more, not less, power to the minority.

Finally, like Condorcet, we assume that information is aggregated only through the decision procedure and not through prior conversation, the news media, and so on. Expanding the model in this direction might show that minority participation can be encouraged not only by electoral procedures but also by better access to the "public sphere."

The first major objection is that a decision procedure is used over time for many different issues: A new procedure is not designed for each issue. A particular person or group might be in a minority over one issue and in a majority over another. The objection then would be that individual preferences and prior beliefs are not known in advance, and hence it is difficult to tailor the procedure to them, an essential premise of the model. Put more strongly, the objection would be that because of the variety of biases over all different possible issues, we do not have a fixed majority and minority, but "cross-cutting majorities" (see, for example, Miller 1983). The problem of minority participation, however, is most acute exactly when there is no such symmetry, when one group of people finds itself in a minority much more often than others. The problem with the cross-cutting majorities argument is not plausibility but applicability. Despite the theoretical compulsion to think of people as homogeneous disembodied spirits, actual groups such as countries have particular histories that often make such presumptions impossible. Belgium's constitution requires cabinet membership to be divided equally between the two language groups, Flemish and French (Fitzmaurice 1983). The 1993 electoral reform in New Zealand retained "dual constituencies" that enabled Maori to vote both as members of Maori districts and as members of "general" districts (Nagel 1995).

The second major objection is that the result here depends on Condorcet's assumption that each person's evidence is equally conclusive. It is true that if the majority thinks the minority's information is poor, then the majority does not care about minority participation; historically, claiming that some people do not know much has been a common justification for their exclusion. The model can still be applied without the equal quality of evidence assumption (see Nitzan and Paroush 1982; Shapley and Grofman 1984) but perhaps not to *democratic* decision making. Democracy's claim that "everyone's opinion counts equally" might mean not only that everyone should be equally considered when balancing between conflicting interests but also that everyone has the same quality information.

The third major objection is that incentive compatibility is an inappropriate assumption: When voting, people do not and should not think instrumentally and selfishly. Instead of a modified procedure to encourage minority participation, under this view the procedure should remain majority rule, and the minority should feel compelled to vote truthfully and sacrifice selfish interests for the common good. But incentive compatibility is in some sense a democratic value. Ballots are secret because of the belief that the voter's reasons are hers alone, not subject to external pressures or motivations such as the common good (but see also Mill 1861). Incentive compatibility assumes the freedom to vote purely in one's own selfish interest.

To summarize, if one accepts Condorcet's argument for majority rule, then one must accept my argument for special minority power. Not doing so involves rejecting either the existence of minority and majority or rejecting incentive compatibility. The existence of minority and majority is defensible on empirical and historical grounds; incentive compatibility is defensible on grounds of freedom.

CONCLUSION

Participation has long been one of democracy's self-proclaimed virtues; the history of the United States, however, is one of exclusion, with each extension of rights laboriously fought for and reluctantly given (Grofman, Handley, and Niemi 1992). Even after women and people of color won the legal right to vote, poll taxes, literacy tests, and other hindrances effectively blocked them from exercising that right. Hindrances to voter registration are still being contested. The 1993 "motor voter" law made it possible to register when applying for a driver's license and at other government offices; previously vetoed by a Republican administration, in the course of its passage, Republican senators tried to weaken it by eliminating the possibility of registering at welfare and unemployment compensation offices ("Goodbye Gridlock . . . " 1993).

The "second generation" of civil rights activism fought against unfair districting, which by diluting minority votes across several districts makes the election of any minority representative almost impossible. What Guinier calls the "third generation" focuses on procedures, such as majority rule, that are used to exclude a minority. In Etowah County, Alabama, redistricting enabled two black commissioners to be elected to the previously all-white county commission in 1986. In response, the four white incumbents eliminated each commissioner's individual authority over his own section of the county and instead made all street repairs subject to majority vote (Greenhouse 1992; Guinier 1994, 8).

The most obvious ground for challenging exclusion and expanding participation is fairness. But perhaps it is not hopelessly idealistic to think in terms of social welfare; Condorcet's information aggregation model lets us do so mathematically. The idea of information aggregation, much more than merely a justification of majority rule, enables us to see democracy not only as the fair resolution of conflicting interests but also as a collective enterprise in which each person's participation benefits everyone else.

APPENDIX

RESULT 2. *Let* $M = \{1, 2, \ldots, m\}$ *be the minority, where* $m \leq (n-1)/2$. *Say that majority beliefs* $\pi_{m+1}(a), \ldots, \pi_n(a) \in (1-q, q)$. *Then there exists* $\delta > 0$ *such that when minority beliefs* $\pi_1(a) = \cdots = \pi_m(a) \in (q, q+\delta)$, *the minority caucus procedure* f_{mc} *is an optimal incentive compatible procedure for a person in the majority, where* f_{mc} *is defined by*

$$f_{mc}(r, a) = \begin{cases} 1 & \textit{if } \#\{i \in N : r_i = a\} \geq (n+1)/2 \\ 1 & \textit{if } \#\{i \in N : r_i = a\} = (n-1)/2 \\ & \textit{and } \#\{i \in M : r_i = a\} \geq k+1 \\ z & \textit{if } \#\{i \in N : r_i = a\} = (n-1)/2 \\ & \textit{and } \#\{i \in M : r_i = a\} = k \\ 0 & \textit{otherwise} \end{cases}$$

where $k \in \{0, 1, \ldots, m\}$, $z \in [0, 1]$ (*if* $k = m$, *then* $z > 0$), *and* $f_{mc}(r, b) = 1 - f_{mc}(r, a)$.

PROOF. For convenience, say $\Omega = \{a, b\}^n$, and for $\omega \in \Omega$, define $\alpha(\omega) = \#\{i \in N : \omega_i = a\}$ and $\beta(\omega) = \#\{i \in N : \omega_i = b\}$. Given $\omega_{-i} \in \{a, b\}^{n-1}$, define $\alpha(\omega_{-i}) = \#\{j \in N \setminus \{i\} : \omega_j = a\}$ and $\alpha_M(\omega_{-i}) = \#\{j \in M \setminus \{i\} : \omega_j = a\}$.

From before, we know that we can write $EU_i(f, s_1, \ldots, s_n) = p(s_1, \ldots, s_n, a, a)\pi_i(a) + p(s_1, \ldots, s_n, b, b)\pi_i(b)$. We know $p(s_1, \ldots, s_n, a, a) = \Sigma_{\omega \in \Omega}\, q^{\alpha(\omega)}(1-q)^{\beta(\omega)} f(s(\omega), a)$, and $p(s_1, \ldots, s_n, b, b) = \Sigma_{\omega \in \Omega}\, q^{\beta(\omega)}(1-q)^{\alpha(\omega)} f(s(\omega), b)$, where $s(\omega) = (s_1(\omega_1), \ldots,$

$s_n(\omega_n))$. But since $f(s(\omega), b) = 1 - f(s(\omega), a)$, we have $EU_i(f, s_1, \ldots, s_n) = \Sigma_{\omega\in\Omega}(\pi_i(a)q^{\alpha(\omega)}(1 - q)^{\beta(\omega)} - \pi_i(b)q^{\beta(\omega)}(1 - q)^{\alpha(\omega)})f(s(\omega), a) + \Sigma_{\omega\in\Omega} \pi_i(b)q^{\beta(\omega)}(1 - q)^{\alpha(\omega)}$. This last term is constant in f and $s_1, \ldots, s_n$, so we can ignore it. If we define $h_i(j) = \pi_i(a)q^j(1 - q)^{n-j} - \pi_i(b)q^{n-j}(1 - q)^j$, we can write $EU_i(f, s_1, \ldots, s_n) = \Sigma_{\omega\in\Omega} h_i(\alpha(\omega))f(s(\omega), a)$. This $h_i(j)$ is simply person i's utility difference from choosing a over b, given that j private experiments support a.

We need some preliminary results on $h_i(j)$. Fact 1 is obvious: $h_1(j) = \cdots = h_m(j)$ (this is because each person in the minority has the same prior beliefs). Since $\pi_i(a) \in (1 - q, q)$ for $i = m + 1, \ldots, n$, we have Fact 2: For $i = m + 1, \ldots, n$, $h_i(j) > 0$ when $j \geq (n + 1)/2$ and $h_i(j) < 0$ when $j \leq (n - 1)/2$ (in other words, each person in the majority wants a to be chosen if a majority of the private experiments support a, and b otherwise). Similarly, we have Fact 3: There exists $\delta > 0$ such that for $\pi_1(a) \in (q, q + \delta)$, for $i = 1, \ldots, m$, we have $h_i(j) > 0$ when $j \geq (n - 1)/2$ and $h_i(j) < 0$ when $j \leq (n - 3)/2$ (in other words, because of her bias toward a, a person in the minority prefers a when the number of private experiments that support a is one less than a majority). Also, we have Fact 4: There exists $\delta > 0$ such that for $\pi_1(a) \in (q, q + \delta)$, we have $h_1((n - 3)/2) + h_1((n - 1)/2) < 0$. This is true because $h_1((n - 3)/2) + h_1((n - 1)/2)$ increases linearly in $\pi_1(a)$ and is negative when $\pi_1(a) = q$. Fact 5 is that there exists $\delta > 0$ such that for $\pi_1(a) \in (q, q + \delta)$, we have $h_n(j) > h_1(j - 1)$ for all $j = 1, \ldots, n$. To show this, first note that $h_i(j)$ increases linearly in $\pi_i(a)$. Since $\pi_n(a) > 1 - q$, we thus know $h_n(j) > q^j(1 - q)^{n-j+1} - q^{n-j+1}(1 - q)^j$. When $\pi_1(a) = q$, we have $h_1(j - 1) = q^j(1 - q)^{n-j+1} - q^{n-j+1}(1 - q)^j$. Since $h_n(j) > h_1(j - 1)$ when $\pi_1(a) = q$, and $h_1(j - 1)$ increases linearly in $\pi_1(a)$, we are done. Finally, we have Fact 6: There exists $\delta > 0$ such that for $\pi_1(a) \in (q, q + \delta)$, we have $h_n(j)/h_n((n - 1)/2) > h_1(j - 1)/h_1((n - 3)/2)$ for $j \leq (n - 3)/2$. To show this, let LHS be the left-hand side of this inequality and RHS be the right-hand side. By taking derivatives and rearranging terms, we can show that LHS strictly increases in $\pi_n(a)$ and RHS strictly increases in $\pi_1(a)$. Also, one can show that when $\pi_n(a) = 1 - q$ and $\pi_1(a) = q$, $LHS = RHS$. Since in fact $\pi_n(a) > 1 - q$, we know that LHS is greater than RHS evaluated at $\pi_1(a) = q$; since RHS is continuous in $\pi_1(a)$, we are done.

Recall that $EU_i(f, s_1, \ldots, s_n) = \Sigma_{\omega\in\Omega} h_i(\alpha(\omega))f(s(\omega), a)$. Thus, an optimal incentive compatible procedure for a person in the majority (say, person n) is a procedure f that maximizes $\Sigma_{\omega\in\Omega} h_n(\alpha(\omega))f(\omega, a)$ subject to the incentive compatibility constraints $\Sigma_{\omega\in\Omega} h_i(\alpha(\omega))f(\omega, a) \geq \Sigma_{\omega\in\Omega} h_i(\alpha(\omega))f(s_i(\omega_i), \omega_{-i}, a)$ for all $i \in N$ and all $s_i \in S$, where $\omega_{-i} = (\omega_1, \ldots, \omega_{i-1}, \omega_{i+1}, \ldots, \omega_n)$.

Recall that it suffices to consider the two deviations s_{aa} (reporting a all the time) and s_{bb} (reporting b all the time). For the deviation s_{aa}, the incentive compatibility constraint is $\Sigma_{\omega\in\Omega} h_i(\alpha(\omega))f(\omega, a) \geq \Sigma_{\omega\in\Omega} h_i(\alpha(\omega))f(a, \omega_{-i}, a)$, that is, $\Sigma_{\omega_{-i}\in\{a,b\}^{n-1}} h_i(\alpha(b, \omega_{-i}))(f(b, \omega_{-i}, a) - f(a, \omega_{-i}, a)) \geq 0$. Similarly, for the deviation s_{bb}, we get $\Sigma_{\omega_{-i}\in\{a,b\}^{n-1}} h_i(\alpha(a, \omega_{-i}))(f(a, \omega_{-i}, a) - f(b, \omega_{-i}, a) \geq 0$. If we define $g(\omega) = f(\omega, a)$, these two incentive constraints are simply

$$\sum_{\omega_{-i}\in\{a,b\}^{n-1}} h_i(\alpha(b, \omega_{-i}))(g(b, \omega_{-i}) - g(a, \omega_{-i})) \geq 0 \quad (\boldsymbol{A_i})$$

$$\sum_{\omega_{-i}\in\{a,b\}^{n-1}} h_i(\alpha(a, \omega_{-i}))(g(a, \omega_{-i}) - g(b, \omega_{-i})) \geq 0. \quad (\boldsymbol{B_i})$$

The optimal incentive compatible procedure for person n is a solution of the following linear programming problem: Choose $\{g(\omega)\}_{\omega\in\Omega}$ to maximize $\Sigma_{\omega\in\Omega} h_n(\alpha(\omega))g(\omega)$ subject to the constraints $A_1, \ldots, A_n, B_1, \ldots, B_n$, and $g(\omega) \in [0, 1]$ for all $\omega \in \Omega$. Let the minority caucus procedure f_{mc} be as defined in the statement of Result 2 and define $g(\omega) = f_{mc}(\omega, a)$. I show that $\{g(\omega)\}_{\omega\in\Omega}$ solves this linear programming problem.

First show that there exists $k \in \{0, 1, \ldots, m\}$ and $z \in [0, 1]$ such that A_1 binds. When $\alpha(\omega_{-1}) \geq (n + 1)/2$ and when $\alpha(\omega_{-1}) \leq (n - 5)/2$, we know that $g(a, \omega_{-1}) = g(b, \omega_{-1})$ and hence $g(b, \omega_{-1}) - g(a, \omega_{-1}) = 0$. So the only terms in A_1 we need to consider are when $\alpha(\omega_{-1}) = (n - 1)/2$ or when $\alpha(\omega_{-1}) = (n - 3)/2$. Using the definition of g, we can write the left-hand side of A_1 as a function of k and z to get $LHS(k, z) = h_1((n - 1)/2) \Sigma_{\alpha(\omega_{-1})=(n-1)/2} ((z - 1)I_{\{\alpha_M(\omega_{-1})=k\}}(\omega_{-1}) + (-1)I_{\{\alpha_M(\omega_{-1})\in\{0,1,\ldots,k-1\}\}}(\omega_{-1})) + h_1((n - 3)/2) \Sigma_{\alpha(\omega_{-1})=(n-3)/2} ((-1)I_{\{\alpha_M(\omega_{-1})\in\{k,\ldots,m-1\}\}}(\omega_{-1}) + (-z)I_{\{\alpha_M(\omega_{-1})=k-1\}}(\omega_{-1}))$, where the I's are indicator functions (for example, $I_{\{\alpha_M(\omega_{-1})=k-1\}}(\omega_{-1}) = 1$ when $\alpha_M(\omega_{-1}) = k - 1$, and 0 otherwise). Note that $LHS(k, 0) = LHS(k + 1, 1)$ and since $h_1((n - 1)/2) > 0$ and $h_1((n - 3)/2) < 0$ from Fact 3, we know that $LHS(k, z)$ decreases in k and increases strictly in z. Define a function $d: [0, m + 1] \to \Re$ as $d(x) = LHS(m - Int(x), x - Int(x))$, where $Int(x)$ is the integer part of x. Since $LHS(k, z)$ is linear in z given k and $LHS(k, 0) = LHS(k + 1, 1)$, we see that $d(x)$ is a continuous piecewise linear function. Since $LHS(k, z)$ decreases in k and increases strictly in z, we know $d(x)$ increases strictly in x. Since $d(0) = LHS(m, 0) < 0$ and $d(m + 1) = LHS(-1, 0) = LHS(0, 1) > 0$, there uniquely exists $x^* \in (0, m + 1)$ such that $d(x^*) = 0$; in other words, there uniquely exists $k \in \{0, 1, \ldots, m\}$ and $z \in [0, 1]$ such that $LHS(k, z) = 0$, that is, A_1 binds.

Note that since $d(m + 1) > 0$, we have $x^* < m + 1$, that is, it cannot be that $k = 0$ and $z = 1$. In other words, we have Fact 7: When $\alpha(\omega) = (n - 1)/2$ and $\alpha_M(\omega) = 0$ (this is possible since $m \leq (n - 1)/2$), we have $g(\omega) < 1$. Also note that $LHS(k, z)$ decreases linearly and strictly in $h_1((n - 1)/2)$ and $h_1((n - 3)/2)$, which in turn increase linearly and strictly in $\pi_1(a)$. So $LHS(k, z)$ decreases linearly and strictly in $\pi_1(a)$. Hence, $d(x)$ decreases linearly and strictly in $\pi_1(a)$, and, hence, x^* increases continuously and strictly in $\pi_1(a)$. Hence, for a fixed k, z increases continuously and strictly in $\pi_1(a)$. When $\pi_1(a) = q$, we have $h_1((n - 1)/2) = 0$, and thus $k = m$ and $z = 0$. Hence, we have Fact 8: For any $\bar{z} > 0$, there exists $\delta > 0$ such that for $\pi_1(a) \in (q, q + \delta)$, we have $k = m$ and $z \in (0, \bar{z})$.

Show that A_n is satisfied and does not bind. Again, we only need to consider when $\alpha(\omega_{-n}) = (n - 1)/2$ and when $\alpha(\omega_{-n}) = (n - 3)/2$. When $\alpha(\omega_{-n}) = (n - 1)/2$, we know that $g(a, \omega_{-n}) = 1$, $g(b, \omega_{-n}) \in \{0, z, 1\}$, and from Fact 2 $h_n(\alpha(b, \omega_{-n})) < 0$; hence, $h_n(\alpha(b, \omega_{-n}))(g(b, \omega_{-n}) - g(a, \omega_{-n})) \geq 0$. When $\alpha(\omega_{-n}) = (n - 3)/2$, we know that $g(a, \omega_{-n}) \in \{0, z, 1\}$, $g(b, \omega_{-n}) = 0$, and from Fact 2 $h_n(\alpha(b, \omega_{-n})) < 0$; hence, $h_n(\alpha(b, \omega_{-n}))(g(b, \omega_{-n}) - g(a, \omega_{-n})) \geq 0$. So $h_n(\alpha(b, \omega_{-n}))(g(b, \omega_{-n}) - g(a, \omega_{-n})) \geq 0$ for all $\omega_{-n} \in \{a, b\}^{n-1}$. When $\alpha(\omega_{-n}) = (n - 1)/2$ and $\alpha_M(\omega_{-n}) = 0$, we know $g(b, \omega_{-n}) < 1$ from Fact 7, $g(a, \omega_{-n}) = 1$, and $h_n(\alpha(b, \omega_{-n})) < 0$, and, hence, $h_n(\alpha(b, \omega_{-n}))(g(b, \omega_{-n}) - g(a, \omega_{-n})) > 0$ for some ω_{-n}. So $\Sigma_{\omega_{-n}\in\{a,b\}^{n-1}} h_n(\alpha(b, \omega_{-n}))(g(b, \omega_{-n}) - g(a, \omega_{-n})) > 0$. Similarly, we can show that $A_{m+1}, \ldots, A_{n-1}$ are satisfied and do not bind.

Show that B_1 is satisfied and does not bind. Again, we only need to consider when $\alpha(\omega_{-1}) = (n - 1)/2$ and when $\alpha(\omega_{-1}) = (n - 3)/2$. When $\alpha(\omega_{-1}) = (n - 1)/2$, we know that $g(a, \omega_{-1}) = 1$, $g(b, \omega_{-1}) \in \{0, z, 1\}$, and $h_1(\alpha(a, \omega_{-1})) > 0$; hence, $h_1(\alpha(a, \omega_{-1}))(g(a, \omega_{-1}) - g(b, \omega_{-1})) \geq 0$. When $\alpha(\omega_{-1}) = (n - 3)/2$, we know that $g(a, \omega_{-1}) \in \{0, z, 1\}$, $g(b, \omega_{-1}) = 0$, and $h_1(\alpha(a, \omega_{-1})) > 0$; hence,

$h_1(\alpha(a, \omega_{-1}))(g(a, \omega_{-1}) - g(b, \omega_{-1})) \geq 0$. So $h_1(\alpha(a, \omega_{-1}))(g(a, \omega_{-1}) - g(b, \omega_{-1})) \geq 0$ for all $\omega_{-1} \in \{a, b\}^{n-1}$. When $\alpha(\omega_{-1}) = (n-1)/2$ and $\alpha_M(\omega_{-1}) = 0$, we know $g(a, \omega_{-1}) = 1$, $g(b, \omega_{-1}) < 1$ from Fact 7, and $h_1(\alpha(a, \omega_{-1})) > 0$; hence, $h_1(\alpha(a, \omega_{-1}))(g(a, \omega_{-1}) - g(b, \omega_{-1})) > 0$ for some ω_{-n}. So $\Sigma_{\omega_{-1} \in \{a,b\}^{n-1}} h_1(\alpha(a, \omega_{-1}))(g(a, \omega_{-1}) - g(b, \omega_{-1})) > 0$.

Finally, show that B_n is satisfied and does not bind. Again, we only need to consider when $\alpha(\omega_{-n}) = (n-1)/2$ and when $\alpha(\omega_{-n}) = (n-3)/2$. If we let LHS be the left-hand side of B_n, we get $LHS = h_n((n+1)/2) \Sigma_{\alpha(\omega_{-n})=(n-1)/2} ((1-z)I_{\{\alpha_M(\omega_{-n})=k\}}(\omega_{-n}) + I_{\{\alpha_M(\omega_{-n}) \in \{0,1,\ldots,k-1\}\}}(\omega_{-n})) + h_n((n-1)/2) \Sigma_{\alpha(\omega_{-n})=(n-3)/2} I_{\{\alpha_M(\omega_{-n}) \in \{k+1,\ldots,m\}\}}(\omega_{-n}) + zI_{\{\alpha_M(\omega_{-1})=k\}}(\omega_{-n}))$. When $k = m$ and $z = 0$, LHS is clearly positive. Also, LHS is continuous in z, so there exists $\bar{z}$ such that for $z \in (0, \bar{z})$, we have $LHS > 0$, that is, B_n is satisfied and does not bind. But from Fact 8, we know there exists $\delta > 0$ such that for $\pi_1(a) \in (q, q + \delta)$, we have $k = m$ and $z \in (0, \bar{z})$. Similarly, we can show that $B_{m+1}, \ldots, B_{n-1}$ are satisfied and do not bind.

We showed above that $A_{m+1}, \ldots, A_n$ and $B_{m+1}, \ldots, B_n$ are satisfied and do not bind. Since everyone in the minority has the same utility function, and since g is symmetric with respect to which member of the minority you are, we know that $B_1, \ldots, B_m$ are satisfied and do not bind, and $A_1, \ldots, A_m$ are satisfied and do bind. So the only constraints we need to consider are $A_1, \ldots, A_m$ and $g(\omega) \in [0, 1]$. Hence we can form the Lagrangian $L = \Sigma_{\omega \in \Omega} h_n(\alpha(\omega))g(\omega) + \Sigma_{i=1}^m \lambda_i(\Sigma_{\omega_{-i} \in \{a,b\}^{n-1}} h_i(\alpha(b, \omega_{-i}))(g(b, \omega_{-i}) - g(a, \omega_{-i}))) + \Sigma_{\omega \in \Omega} (\mu(\omega) - \nu(\omega))g(\omega)$. By the Kuhn-Tucker theorem, if we can find nonnegative $\lambda_1, \ldots, \lambda_m$, $\{\mu(\omega)\}_{\omega \in \Omega}$, $\{\nu(\omega)\}_{\omega \in \Omega}$ such that $\partial L/\partial g(\omega) = 0$ for all $\omega \in \Omega$, where $\mu(\omega) = 0$ if $g(\omega) > 0$ and $\nu(\omega) = 0$ if $g(\omega) < 1$, then g is an optimal incentive compatible procedure.

Define $\lambda = h_n((n-1)/2)/(kh_1((n-3)/2) - (m-k)h_1((n-1)/2)))$. We know $\lambda > 0$ because $h_n((n-1)/2) < 0$, $h_1((n-3)/2) < 0$, and $h_1((n-1)/2) > 0$ from facts 2 and 3; also, from Fact 5 we know $h_n((n-1)/2) > h_1((n-3)/2)$, and, hence, $\lambda < h_n((n-1)/2)/(kh_1((n-3)/2)) < 1/k$. Let $\lambda_1 = \cdots = \lambda_m = \lambda$. Recalling Fact 1, we get $\partial L/\partial g(\omega) = h_n(\alpha(\omega)) + \lambda(-\alpha_M(\omega)h_1(\alpha(\omega) - 1) + (M - \alpha_M(\omega))h_1(\alpha(\omega))) + \mu(\omega) - \nu(\omega)$, where $\alpha_M(\omega) = \#\{i \in M: \omega_i = a\}$.

We need to show $\partial L/\partial g(\omega) = 0$ for all $\omega \in \Omega$, that is, $h_n(\alpha(\omega)) = \lambda(\alpha_M(\omega)h_1(\alpha(\omega) - 1) - (m - \alpha_M(\omega))h_1(\alpha(\omega))) - \mu(\omega) + \nu(\omega)$. Let $RHS = \lambda(\alpha_M(\omega)h_1(\alpha(\omega) - 1) - (m - \alpha_M(\omega))h_1(\alpha(\omega)))$. Note that for ω such that $g(\omega) = 1$, we have $\mu(\omega) = 0$, and hence it suffices to show that $h_n(\alpha(\omega)) \geq RHS$. Similarly, for ω such that $g(\omega) = 0$, we have $\nu(\omega) = 0$, and it suffices to show that $h_n(\alpha(\omega)) \leq RHS$. Finally, for ω such that $g(\omega) \in (0, 1)$, we have $\mu(\omega) = \nu(\omega) = 0$, and it suffices to show that $h_n(\alpha(\omega)) = RHS$.

Say $\alpha(\omega) = (n-1)/2$. When $\alpha_M(\omega) = k$, we have $g(\omega) = z \in [0, 1]$; it is obvious that $h_n((n-1)/2) = RHS$ from our definition of λ above. We can write $RHS = \lambda(\alpha_M(\omega)(h_1((n-3)/2) + h_1((n-1)/2)) - mh_1((n-1)/2))$. Since $h_1((n-3)/2) + h_1((n-1)/2) < 0$ from Fact 4, we know that RHS decreases with $\alpha_M(\omega)$. Hence, when $\alpha_M(\omega) \geq k + 1$ (and $g(\omega) = 1$), we have $h_n((n-1)/2) \geq RHS$; when $\alpha_M(\omega) \leq k - 1$ (and $g(\omega) = 0$), we have $h_n((n-1)/2) \leq RHS$.

Say $\alpha(\omega) \geq (n+1)/2$ and thus $g(\omega) = 1$. From Fact 5, we know $h_n(\alpha(\omega)) > h_1(\alpha(\omega) - 1)$. Since $\lambda < 1/k$, and since we showed before that there exists r such that for $\pi_1(a) \in (q, r)$ we have $k = m$, we can assume that $\lambda < 1/m$, and hence $\lambda m < 1$. Thus $h_n(\alpha(\omega)) > \lambda m h_1(\alpha(\omega) - 1) \geq RHS$, since by Fact 3 $h_1(\alpha(\omega) - 1) > 0$ and $h_1(\alpha(\omega)) > 0$.

Say $\alpha(\omega) \leq (n-3)/2$, and thus $g(\omega) = 0$. From Fact 6, we know that $h_n(\alpha(\omega))/h_n((n-1)/2) > h_1(\alpha(\omega) - 1)/h_1((n-3)/2)$. Since $h_n((n-1)/2) < 0$, we have $h_n(\alpha(\omega)) < (h_n((n-1)/2)/h_1((n-3)/2))h_1(\alpha(\omega) - 1)$. Since we have $k = m$, we can write $h_n(\alpha(\omega)) < (1/k)(h_n((n-1)/2)/h_1((n-3)/2))mh_1(\alpha(\omega) - 1)$. But from our definition of λ above, it is easy to see that $\lambda \leq (1/k)(h_n((n-1)/2)/h_1((n-3)/2))$. Since $h_1(\alpha(\omega) - 1) < 0$ and $h_1(\alpha(\omega)) < 0$ by Fact 3, we have $h_n(\alpha(\omega)) < \lambda m h_1(\alpha(\omega) - 1) < RHS$. *Q.E.D.*

RESULT 3. *Let* M = {1, 2, ..., m} *be the minority, where* m ≤ (n − 1)/2. *Say that majority beliefs* $\pi_{m+1}(a), \ldots, \pi_n(a) \in (1 - q, q)$. *Then there exists* $\delta > 0$ *such that when minority beliefs* $\pi_1(a) = \cdots = \pi_m(a) \in (q, q + \delta)$, *for any optimal incentive compatible procedure* f, *we have*:

(i) f(r, a) = 1 *for all* r *such that* $\#\{i \in N: r_i = a\} \geq (n+1)/2$;
(ii) f(r, a) = 0 *for all* r *such that* $\#\{i \in N: r_i = a\} \leq (n-3)/2$;
(iii) f(r, a) > 0 *for some* r *such that* $\#\{i \in N: r_i = a\} = (n-1)/2$.

PROOF. Result 2 says that f_{mc} is an optimal incentive compatible procedure; as before, let $g(\omega) = f_{mc}(\omega, a)$. From Fact 8, we can assume that $k = m$ and $z \in (0, 1)$. To show (i) and (ii) above, say that f^* is another optimal incentive compatible procedure, and let $g^*(\omega) = f^*(\omega, a)$. Assume there exists ω^* such that either $g^*(\omega^*) < 1$ and $\alpha(\omega^*) \geq (n+1)/2$ or $g^*(\omega^*) > 0$ and $\alpha(\omega^*) \leq (n-3)/2$. Define $g'(\omega) = (g(\omega) + g^*(\omega))/2$. We know that $g'(\omega^*) \in (0, 1)$ and also that $g'(\omega') \in (0, 1)$ for some ω' such that $\alpha(\omega') = (n-1)/2$ and $\alpha_M(\omega') = m$.

By convexity we know that g' is also an optimal incentive compatible procedure. Since we know that the constraints $A_{m+1}, \ldots, A_n, B_1, \ldots, B_n$ do not bind for g, we know that they do not bind for g'. Hence we can form the Lagrangian $L = \Sigma_{\omega \in \Omega} h_n(\alpha(\omega))g'(\omega) + \Sigma_{i=1}^m \lambda_i(\Sigma_{\omega_{-i} \in \{a,b\}^{n-1}} h_i(\alpha(b, \omega_{-i}))(g'(b, \omega_{-i}) - g'(a, \omega_{-i}))) + \Sigma_{\omega \in \Omega}(\mu(\omega) - \nu(\omega))g'(\omega)$. By the Kuhn-Tucker theorem, there must exist nonnegative $\lambda_1, \ldots, \lambda_m$, $\{\mu(\omega)\}_{\omega \in \Omega}$, $\{\nu(\omega)\}_{\omega \in \Omega}$ such that $\partial L/\partial g'(\omega) = 0$ for all $\omega \in \Omega$, where $\mu(\omega) = 0$ if $g(\omega) > 0$ and $\nu(\omega) = 0$ if $g(\omega) < 1$.

Since $g'(\omega') \in (0, 1)$, we have $\mu(\omega') = \nu(\omega') = 0$, and, hence, $h_n(\alpha(\omega')) = \Sigma_{\{i \in M: \omega'_i = a\}} \lambda_i h_i(\alpha(\omega') - 1) - \Sigma_{\{i \in M: \omega'_i = b\}} \lambda_i h_i(\alpha(\omega'))$. Since $\alpha(\omega') = (n-1)/2$ and $\omega'_i = a$ for all $i \in M$, we have $h_n((n-1)/2) = (\Sigma_{i \in M} \lambda_i)h_1((n-3)/2)$. Hence, $\Sigma_{i \in M} \lambda_i = h_n((n-1)/2)/h_1((n-3)/2) < 1$ by Fact 5 and because $h_1((n-3)/2) < 0$.

Similarly, since $g'(\omega^*) \in (0, 1)$, we have $\mu(\omega^*) = \nu(\omega^*) = 0$, and thus $h_n(\alpha(\omega^*)) = \Sigma_{\{i \in M: \omega^*_i = a\}} \lambda_i h_i(\alpha(\omega^*) - 1) - \Sigma_{\{i \in M: \omega^*_i = b\}} \lambda_i h_i(\alpha(\omega^*))$. If $\alpha(\omega^*) \geq (n+1)/2$, we know that $h_i(\alpha(\omega^*) - 1) > 0$ and $h_i(\alpha(\omega^*)) > 0$ for $i \in M$, and hence $h_n(\alpha(\omega^*)) \leq \Sigma_{i \in M} \lambda_i h_i(\alpha(\omega^*) - 1) = (\Sigma_{i \in M} \lambda_i)h_1(\alpha(\omega^*) - 1)$. Since $\Sigma_{i \in M} \lambda_i < 1$, we have $h_n(\alpha(\omega^*)) < h_1(\alpha(\omega^*) - 1)$. But this contradicts Fact 5. If $\alpha(\omega^*) \leq (n-3)/2$, we know that $h_i(\alpha(\omega^*) - 1) < 0$ and $h_i(\alpha(\omega^*)) < 0$ for $i \in M$, and, hence, $h_n(\alpha(\omega^*)) \geq \Sigma_{i \in M} \lambda_i h_i(\alpha(\omega^*) - 1) = (\Sigma_{i \in M} \lambda_i)h_1(\alpha(\omega^*) - 1)$. By Fact 6, we know that $h_n(\alpha(\omega^*)) < (h_n((n-1)/2)/h_1((n-3)/2))h_1(\alpha(\omega^*) - 1) = (\Sigma_{i \in M} \lambda_i)h_1(\alpha(\omega^*) - 1)$, a contradiction.

Thus (i) and (ii) above are true. If $f(r, a) = 0$ for all r such that $\#\{i \in N: r_i = a\} = (n-1)/2$, then it is easy to see that constraints $A_1, \ldots, A_m$ are violated; hence we have (iii). *Q.E.D.*

RESULT 4. *Let* M = {1, 2, ..., m} *be the minority. Say that majority beliefs* $\pi_{m+1}(a), \ldots, \pi_n(a) \in [1/2, q)$. *Then there* *exists* $\delta > 0$ *such that when minority beliefs* $\pi_1(a), \ldots, \pi_m(a) \in (q, q + \delta)$, *the procedure* f_{an} *is the unique optimal anonymous incentive compatible procedure for a person in the majority, where* f_{an} *is defined by*

$$f_{an}(r, a) = \begin{cases} 1 & \text{if } \#\{i \in N : r_i = a\} \geq (n + 1)/2 \\ z & \text{if } \#\{i \in N : r_i = a\} = (n - 1)/2 \\ 0 & \text{otherwise} \end{cases}$$

where $z \in (0, 1)$ *and* $f_{an}(r, b) = 1 - f_{an}(r, a)$.

PROOF. Without loss of generality we can assume $\pi_1(a) = \cdots = \pi_{\bar{m}}(a) > \pi_{\bar{m}+1}(a) \geq \cdots \geq \pi_m(a)$; in other words, persons $1, \ldots, \bar{m}$ are the most biased members of the minority. Since we consider only anonymous procedures, we can write $f_{an}(\omega, a) = \gamma(\alpha(\omega))$. Hence we can write $EU_i(f_{an}, id, \ldots, id) = \sum_{j=0}^{n} l(j)h_i(j)\gamma(j)$, where $l(j) = \#\{\omega \in \Omega : \alpha(\omega) = j\} = n!/(j!(n - j)!)$. Similarly we can write constraint A_i as $\sum_{j=0}^{n} l_b(j)h_i(j)(\gamma(j) - \gamma(j + 1)) \geq 0$, where $l_b(j) = \#\{\omega \in \Omega : \omega_i = b \text{ and } \alpha(\omega) = j\} = (n - 1)!/(j!(n - j - 1)!)$, and constraint B_i as $\sum_{j=0}^{n} l_a(j)h_i(j)(\gamma(j) - \gamma(j - 1)) \geq 0$, where $l_a(j) = \#\{\omega \in \Omega : \omega_i = a \text{ and } \alpha(\omega) = j\}$.

Now we can proceed in a very similar fashion to the proof of Result 2. We have $\gamma(j) = 1$ if $j \geq (n + 1)/2$, $\gamma(j) = z$ if $j = (n - 1)/2$, and $\gamma(j) = 0$ if $j \leq (n - 3)/2$. Show that there exists $z \in (0, 1)$ such that A_1 binds. We can write the left-hand side of A_1 as a function of z to get $LHS(z) = l_b((n - 1)/2)h_1((n - 1)/2)(z - 1) + l_b((n - 3)/2)h_1((n - 3)/2)(-z)$. Since $h_1((n - 1)/2) > 0$ and $h_1((n - 3)/2) < 0$ by Fact 3, we know that $LHS(z)$ increases linearly and strictly in z. We know $LHS(0) < 0$ and $LHS(1) > 0$, and hence there exists $z \in (0, 1)$ such that $LHS(z) = 0$, that is, A_1 binds.

Also note that $LHS(z)$ decreases linearly and strictly in $h_1((n - 1)/2)$ and $h_1((n - 3)/2)$, which in turn increase linearly and strictly in $\pi_1(a)$. So $LHS(z)$ decreases linearly and strictly in $\pi_1(a)$. Hence the $z \in (0, 1)$ which satisfies $LHS(z) = 0$ increases continuously and strictly in $\pi_1(a)$. When $\pi_1(a) = q$, we have $h_1((n - 1)/2) = 0$, and thus $z = 0$. Hence we have Fact 8a: For any $\bar{z} > 0$, there exists $\delta > 0$ such that for $\pi_1(a) \in (q, q + \delta)$, we have $z \in (0, \bar{z})$.

The constraints $A_2, \ldots, A_{\bar{m}}$ are identical to A_1. Since $LHS(z)$ above decreases strictly in $\pi_1(a)$, it is easy to see that constraints $A_{\bar{m}+1}, \ldots, A_m$ are satisfied and do not bind.

Show that A_n is satisfied and does not bind. The left-hand side of A_n is simply $l_b((n - 1)/2)h_n((n - 1)/2)(z - 1) + l_b((n - 3)/2)h_n((n - 3)/2)(-z)$, which is positive since $h_n((n - 1)/2) < 0$ and $h_n((n - 3)/2) < 0$ by Fact 2 and $z \in (0, 1)$. Similarly, constraints $A_{m+1}, \ldots, A_{n-1}$ are satisfied and do not bind.

Show that B_1 is satisfied and does not bind. The left-hand side of B_1 is simply $l_a((n + 1)/2)h_1((n + 1)/2)(1 - z) + l_a((n - 1)/2)h_1((n - 1)/2)(z)$, which is positive since $h_1((n + 1)/2) > 0$ and $h_1((n - 1)/2) > 0$ by Fact 3 and $z \in (0, 1)$. Similarly, constraints $B_2, \ldots, B_m$ are satisfied and do not bind.

Show that B_n is satisfied and does not bind. The left-hand side of B_n is simply $LHS = l_a((n + 1)/2)h_n((n + 1)/2)(1 - z) + l_a((n - 1)/2)h_n((n - 1)/2)(z)$. When $z = 0$, since $h_n((n + 1)/2) > 0$ by Fact 2, LHS is positive. Since LHS is continuous in z, there exists $\bar{z}$ such that for $z \in (0, \bar{z})$, we have $LHS > 0$, that is, B_n is satisfied and does not bind. But from Fact 8a, we know there exists r such that for $\pi_1(a) \in (q, r)$, we have $z \in (0, \bar{z})$. Similarly, constraints $B_{m+1}, \ldots, B_{n-1}$ are satisfied and do not bind.

So the only constraints on $\gamma(j)$ we need to consider are A_1 and $\gamma(j) \in [0, 1]$. Hence we can form the Lagrangian $L = \sum_{j=0}^{n} l(j)h_n(j)\gamma(j) + \lambda(\sum_{j=0}^{n} l_b(j)h_1(j)(\gamma(j) - \gamma(j + 1))) + \sum_{j=0}^{n} (\mu(j) - \nu(j))\gamma(j)$. By the Kuhn-Tucker theorem, if we can find nonnegative λ and $\{\mu(j)\}_{j=0}^{n}$, $\{\nu(j)\}_{j=0}^{n}$ such that $\partial L/\partial\gamma(j) = 0$, where $\mu(j) = 0$ if $\gamma(j) > 0$ and $\nu(j) = 0$ if $\gamma(j) < 1$, then $\gamma(j)$ is an optimal anonymous incentive compatible procedure.

Define $\lambda = l((n - 1)/2)h_n((n - 1)/2)/(l_b((n - 3)/2)h_1((n - 3)/2) - l_b((n - 1)/2)h_1((n - 1)/2)))$. We know $\lambda > 0$ because $h_n((n - 1)/2) < 0$, $h_1((n - 3)/2) < 0$, and $h_1((n - 1)/2) > 0$ from facts 2 and 3. We also know Fact 9: $\lambda < l((n - 1)/2)h_n((n - 1)/2)/(l_b((n - 3)/2)h_1((n - 3)/2))$.

We need to show $\partial L/\partial\gamma(j) = 0$ for all $j = 0, \ldots, n$, that is, $l(j)h_n(j) = \lambda(l_b(j - 1)h_1(j - 1)) - l_b(j)h_1(j)) - \mu(j) + \nu(j)$. Let $RHS = \lambda(l_b(j - 1)h_1(j - 1) - l_b(j)h_1(j))$. Note that when $\gamma(j) = 1$, we have $\mu(j) = 0$, and hence it suffices to show that $l(j)h_n(j) \geq RHS$; when $\gamma(j) = 0$, we have $\nu(\omega) = 0$, and it suffices to show that $l(j)h_n(j) \leq RHS$; finally, when $\gamma(j) \in (0, 1)$, we have $\mu(\omega) = \nu(\omega) = 0$, and it suffices to show that $l(j)h_n(j) = RHS$.

When $j = (n - 1)/2$, we have $\gamma(j) = z \in (0, 1)$, and it is obvious that $l(j)h_n(j) = RHS$ from our definition of λ above.

When $j \leq (n - 3)/2$, we have $\gamma(j) = 0$. By simplifying terms, $l((n - 1)/2)l_b(j - 1)/(l(j)l_b((n - 3)/2)) = 2j/(n - 1) < 1$ since $j \leq (n - 3)/2$. Hence, $l(j) > l((n - 1)/2)l_b(j - 1)/l_b((n - 3)/2)$. From Fact 6 and since $h_n((n - 1)/2) < 0$ we know that $h_n(j) < h_n((n - 1)/2)h_1(j - 1)/h_1((n - 3)/2)$. Hence, we can "multiply" these inequalities to get $l(j)h_n(j) < [l((n - 1)/2)h_n((n - 1)/2)/(l_b((n - 3)/2)h_1((n - 3)/2))]l_b(j - 1)h_1(j - 1)$. But using Fact 9, we get $l(j)h_n(j) < \lambda l_b(j - 1)h_1(j - 1) < RHS$ since $h_1(j) < 0$.

When $j \geq (n + 1)/2$, we have $\gamma(j) = 1$. We now need the following fact. Fact 10: There exists $\delta > 0$ such that for $\pi_1(a) \in (q, q + \delta)$ and $\pi_n(a) \in [1/2, q)$, $h_n(j)h_1((n - 3)/2)/(h_n((n - 1)/2))h_1(j - 1)) \geq 2(2j - n)/(2j - n - 1)$ for $j \geq (n + 1)/2$. To prove this, first assume that $\pi_1(a) = q$ and $\pi_n(a) = 1/2$. By expanding terms it is not hard to show that $h_1((n - 3)/2)/h_n((n - 1)/2) = 2$. As q approaches 1/2, by L'Hopital's rule we can show that $h_n(j)/h_1(j - 1)$ approaches $(2j - n)/(2j - n - 1)$. With much computation, we find that $\partial(h_n(j)/h_1(j - 1))/\partial q = (q(1 - q))^{n-1}/(2h_1(j - 1)^2)((2j - n)(1 - 2q) + q^{2j-n+1}(1 - q)^{n-2j+1} - q^{n-2j+1}(1 - q)^{2j-n+1})$. Hence, $\partial(h_n(j)/h_1(j - 1))/\partial q$ has the same sign as $(2j - n)(1 - 2q) + q^{2j-n+1}(1 - q)^{n-2j+1} - q^{n-2j+1}(1 - q)^{2j-n+1}$, which is strictly positive for $q > 1/2$ (because it is 0 at $q = 1/2$, and its derivative with respect to q is strictly positive for $q > 1/2$). Since $h_n(j)/h_1(j - 1)$ is strictly increasing in q and approaches $(2j - n)/(2j - n - 1)$ as q approaches 1/2, since $q > 1/2$, we know $h_n(j)/h_1(j - 1) > (2j - n)/(2j - n - 1)$. Hence, $h_n(j)h_1((n - 3)/2)/(h_n((n - 1)/2))h_1(j - 1)) \geq 2(2j - n)/(2j - n - 1)$ when $\pi_1(a) = q$ and $\pi_n(a) = 1/2$. Since $h_n(j)h_1((n - 3)/2)/(h_n((n - 1)/2))h_1(j - 1))$ increases in $\pi_n(a)$ (we can show this by taking its derivative) and is continuous in $\pi_1(a)$, we are done.

Since $j \geq (n + 1)/2$ and $j \leq n$, it is easy to show that $2(n - 2j)/(1 + n - 2j) \geq 2j/(n - 1)$. Hence, by Fact 10 we have $h_n(j)h_1((n - 3)/2)/(h_n((n - 1)/2))h_1(j - 1)) \geq 2j/(n - 1) = l((n - 1)/2)l_b(j - 1)/(l(j)l_b((n - 3)/2))$. Since $h_n((n - 1)/2) < 0$, $h_1((n - 3)/2) < 0$, and $h_1(j - 1) > 0$, we have $l(j)h_n(j) \geq [l((n - 1)/2)h_n((n - 1)/2)/(l_b((n - 3)/2)h_1((n - 3)/2))]l_b(j - 1)h_1(j - 1)$. But using Fact 9, we get $l(j)h_n(j) \geq \lambda l_b(j - 1)h_1(j - 1) > RHS$ since $h_1(j) > 0$.

To show that $\gamma(j)$ is the unique anonymous incentive compatible procedure, let $\gamma^*(j)$ be another anonymous incentive compatible procedure. Say that $\gamma^*(j)$ satisfies $\gamma^*(j) = 0$ when $j \leq (n-3)/2$ and $\gamma^*(j) = 1$ for $j \geq (n+1)/2$. If $\gamma^*((n-1)/2) = 0$, then it is easy to show that constraint A_1 is violated. If $\gamma^*((n-1)/2) = 1$, then it is easy to show that constraint B_n is violated. If $\gamma^*((n-1)/2) \in (0, 1)$, it is easy to show that $\gamma^* = \gamma$. So it must be that there exists $j^* \neq (n-1)/2$ such that $\gamma^*(j^*) \in (0, 1)$. Hence, if we define $\gamma'(j) = (\gamma(j) + \gamma^*(j))/2$, we have $\gamma'((n-1)/2) \in (0, 1)$ and $\gamma'(j^*) \in (0, 1)$. By convexity, we know that γ' must also be an anonymous incentive compatible procedure. Since we showed earlier that the incentive compatibility constraints $A_{\bar{m}+1}, \ldots, A_n$ and $B_1, \ldots, B_n$ do not bind for $\gamma(j)$, they also do not bind for $\gamma'(j)$. Hence, the only possible binding incentive compatibility constraint is A_1, and we can form the Lagrangian $L = \Sigma_{j=0}^{n} l(j)h_n(j)\gamma'(j) + \lambda'(\Sigma_{j=0}^{n} l_b(j)h_1(j)(\gamma'(j) - \gamma'(j+1))) + \Sigma_{j=0}^{n}(\mu'(j) - \nu'(j))\gamma'(j)$. By the Kuhn-Tucker theorem there must exist $\lambda' > 0$ and $\{\mu'(j)\}_{j=0}^{n}$, $\{\nu'(j)\}_{j=0}^{n}$ such that $\partial L/\partial\gamma'(j) = 0$, where $\mu'(j) = 0$ if $\gamma'(j) > 0$ and $\nu'(j) = 0$ if $\gamma'(j) < 1$. As we showed earlier, $\partial L/\partial\gamma'(j) = 0$ is equivalent to $l(j)h_n(j) = RHS' - \mu'(j) + \nu'(j)$, where $RHS' = \lambda'(l_b(j-1)h_1(j-1) - l_b(j)h_1(j))$. Since $\gamma'((n-1)/2) \in (0, 1)$, we have $\mu'((n-1)/2) = \nu'((n-1)/2) = 0$, and, hence, we must have $\lambda' = \lambda l((n-1)/2)h_n((n-1)/2)/(l_b((n-3)/2)h_1((n-3)/2) - l_b((n-1)/2)h_1((n-1)/2)))$. Since also $\gamma'(j^*) \in (0, 1)$, we must have $l(j^*)h_n(j^*) = RHS' = RHS$; this is a contradiction because we showed earlier that $l(j)h_n(j) < RHS$ for $j \leq (n-3)/2$ and $l(j)h_n(j) > RHS$ for $j \geq (n+1)/2$. *Q.E.D.*

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